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FUELS OF THE FUTURE *

BY

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There are some subjects of universal significance which give endless opportunities to speakers and writers, but because responsibility for them is widely spread, effective action is rarer than solemn pronouncement. The supply of energy in the future is one of these topics, in relation to the expansion of our nation's needs and the gradual exhaustion of some of our principal natural sources. The facts are frequently presented in terms which are intended to cause alarm, but as it is unnatural to be alarmed about a calamity which is expected to occur several generations hence, it is not surprising that the warnings have only a gradual and not a sobering effect. But the subject of energy sources can be shown to bear directly on contemporary work and thought, and opportunities for present-day action can often be indicated. Never before, surely, have research workers and engineers devoted so much attention to methods and techniques which are not likely to be commercially suitable for some years. Nuclear energy, solar energy, wind power, tidal power, fuel cells—may make a useful contribution when relative costs of different energy sources are changing slowly, and the change gives rise to opportunities in the design of new plant and equipment, and in many other ways. From the standpoint of business, an enormous and accelerated rate of capital investment will give expression to these changes. The engineer's interpretation is that the changes will demand more training, adequate technical and scientific knowledge, and profound research and development work. The next 50 years or so will present a great challenge to men of foresight, courage, adaptability and creativeness since it will be a time of realization that there is scope for new de-

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signs, new applications and greater efficiency toward an abundant supply of power.

The expansion of present conventional fuel-burning facilities must continue until the use of atomic fuels is sufficiently developed to assure a definite place in the power generating industry. We should also be reminded that the conventional fuel-burning power plants which are at present in use, and which will be built in the years ahead, will be employed for their normal life expectancy since it is not economically possible to discard or abandon investments in plants which are in good, serviceable condition. Even though we have no exact knowledge concerning the relative competitive position of fuels in the future, it is well to realize that our resources of oil and gas are being quite rapidly depleted and that the requirements of these fuels for higher uses is increasing rapidly. We can expect, therefore, that coal will supply an ever increasing portion of the energy necessary for power generation. It will thus be observed that the possible adverse effects of atomic fuels on the coal industry would appear to be quite far in the future. Our uses of petroleum and natural gas for purposes other than power generation will continue to increase.

In the past the population of the United States has grown quite rapidly and the gross output of the nation has grown even faster. It may be more than a coincidence that we have had abundant low-cost energy at this period in our development, while other nations who were in a position to compete commercially with us had higher energy costs to contend with and less lively economies.[2]

Some 72 years ago, the first commercial installation for the generation and distribution of electric power was placed in operation in New York City. In the relatively short time since, the business of producing and selling electrical energy has grown into one of the country's principal industries. It represents net capital in excess of twenty billions of dollars and an annual gross revenue of about six billion. It is hard to visualize the profound change in the mode of living that has taken place within one man's life span which is mainly due to electricity. Most of the things that have made our present mode of life possible could not have been produced without electric power and the great increase of productivity per man is mainly due to this power. Growth of the electric-power industry in the early days was slow but by 1910 there was a total installed capacity of about 4,800,000 kw. and the annual production approximated 14 billion kwhr. This was about 150 kwhr. per capita. Based on reports to the Federal Power Commission including last September, the total production of electric power for the year by utility and industrial plants will exceed 510 billion kwhr. This is equivalent to an average hourly load throughout the year of more than 58 million kw. or 3100 kwhr. per capita.

[2] Palmer Putnam, "Energy in the Future," New York, D. Van Nostrand Co., 1953.

The growth of the electric-power industry has been accompanied by an almost fantastic development in the technique of its production.[3] Demand for electric energy has been the inspiration for rapid improvement of means for both its production and utilization. Greater availability and lower costs have fostered greater demand. In the early 90's a kilowatt-hour cost the customer upwards of twenty cents and today the national average domestic rate is about three cents. It probably can be said that the development of electric-power production never could have gone very far except for the invention and development of the stream turbine. Nor would the development of the steam turbine have reached its present status without the demand created by electric-power generation. Top steam pressure which was about 300 psi. in 1925 rose steadily to the present 2400 psi., and an installation which will employ a throttle pressure of 4500 psi. now is being planned for completion in the next few years. Use of these higher pressures was made possible only through the development of welding techniques to a point where riveted construction could be eliminated and steam-generator drums be entirely welded. Steam temperatures were limited in the early days by the materials of construction then available. Intensive research, specifically in development of new alloys and improved manufacturing and treating techniques, have permitted increases in steam temperature until today 1000° F. is common practice, 1050° F. becoming more common, and 1150° or 1200° F. seem to be "just around the corner."

Along with increases in steam pressure and temperature has come a steady increase in capacity of generating units. The largest unit now planned for installation will have a stated maximum capability of 260,000 kw. This unit will operate with throttle pressure of 2000 psi., a total temperature of 1050° F., and reheat to 1000° F.

The United States has been in the forefront of electric power development for a long period. Thanks to the genius of one of our great pioneers, Thomas Edison, electric power was born as an industry in this country just 75 years ago. One of the hazards of leadership is the tendency to grow complacent, to let younger, more vigorous competitors pull ahead while the original leader coasts in his earlier successes. We are fortunate in that no such complacency has developed in relation to electric power here. Evidence of our enterprise in this respect is apparent from many sources and nowhere better than from an examination of the world-wide expansion in electric power production that has taken place in the period 1940–1952. Canada's production expanded 106 per cent, France's 114 per cent, that of the United Kingdom 117 per cent, and that of Soviet Russia—with all its series of five-year plans—150 per cent. In the same interval, the United States increased its power production 180 per cent.

[3] The author is indebted to Mr. Philip Sporn, President of the American Gas and Electric Service Corp., for the comparison of electric and atomic power.

The source of this electric energy used in the United States is also significant. While water power has played a significant role in the history of electric power development in this country, the size of its role has been decreasing. In 1953 the source of electric energy was roughly 24 per cent hydro and 76 per cent fuels. Only eight years earlier, hydro accounted for 30 per cent of the total. It is quite certain that by 1978 the per cent generated by hydro is going to be less than 10 per cent. The balance will be supplied by fuel—most of it by the fossil fuels, coal, oil, and gas; and some relatively small part, perhaps, by atomic fuel.

In recent years, oil and gas have made inroads in the fuel market theretofore enjoyed by coal. In the long-term picture, however, these are transient occurrences. As oil and gas supplies decline, coal will again pick up more and more the fuel burden dropped by them.

Do we have adequate coal reserves that can be economically burned to supply our energy needs if they are to come from conventional fuels? There is no agreement even among geologic experts as to what these reserves are. But it is well to note that disagreement is as to whether the reserves are adequate for the next 2000 years or only for the next 300 years. If recent judgment has veered toward the lower figure, at least there is no disagreement that reserves adequate for the next several hundred years are available and have been proven.

Now, what is the status of energy production by atomic fuel? I am sure you all know that when we talk about atomic power, we are talking about power to be produced by presently known means and technologies, but substituting uranium-235 or plutonium or some day, perhaps, uranium-233 for coal or oil or gas. That is, electric power will continue to be generated mostly by steam turbines in which heat energy will be utilized to generate steam, which will drive turbines, which in turn will drive the electric generators—basically the same kind of generators we have today—to produce the electric power. The chief difference is that the fuel will be atomic fuel.

It is very important to avoid any confusion about the technical feasibility of atomic power. Atomic power in that sense is here; that is, we can produce power from the fissioning of atomic material—natural uranium, or U-235 enriched uranium, or plutonium. Technically this has already been accomplished. A reactor adequate to drive an advanced design submarine (*Nautilus*) has been completed. The keel of a second submarine, the *USS Sea Wolf*, has been laid; and it, too, will be driven by an atomic power plant of a somewhat different design than that which powers the *Nautilus*. Other reactors for the propulsion of naval war vessels are in process of development. Any one of these reactors could be put to work to generate power for peacetime use in the home, on the farm, and in industry, except for one thing—the item of economics.

As the Chairman of the Atomic Energy Commission put the matter very succinctly in his remarks of some months ago:

"The naval atomic energy units do not have to meet present competition with diesel fuels, of course, because of other great tactical advantages of atomic power plants. These include the ability to operate submerged for long intervals, and to cruise at speed for many thousands of miles without refueling. On the other hand, civilian atomic power must justify itself by being competitive in cost with present power sources."

So we see that, while atomic power has been proved, atomic power that can take its position in the market place and meet the test of competition with other forms of energy is another matter.[4]

In general, however, while the cost of power from atomic energy power plants, as now conceived, is undeniably high, historically the trend in a new industry is for a rapid reduction in cost as the technology develops and volume increases. On the other hand, in an old and established industry, costs tend to stablize and eventually even to begin to rise again as the technology is perfected and rising labor costs begin to dominate. Many in the know of the atomic energy business feel that the cross-over point on these two curves is likely to come somewhere between 5 and 15 years from now depending on the vigor and intensity of the development effort.

A number of impressive foreign projects are under way: In England and in Norway, reactors are in process of construction having as their objective the development of atomic power for ordinary civilian use. The last session of Congress authorized the Atomic Energy Commission to spend up to $7 million appropriated for research and development for the beginning of construction of a land-based reactor also for ordinary electric power production. But no one who is familiar with power reactor technology believes that this reactor—or any other first reactor—would be able to produce electric power that approaches in cost, power produced by more conventional means. Why is this so? Very simply: There are still a great many technical problems to be solved in reactor design, in reactor vessels—that is, the shells or drums housing the reactors—in design of atomic fuel elements, heat transfer systems, fuel element life, fuel reprocessing, valving, controls (the list could be greatly extended) before we can bring the cost of reactors and atomic fuel down to the point where the energy they produce is competitive with conventionally produced electric power. This is especially true in most of the economically important areas of the United States where abundant and economical fuel supplies are available.

In attempting to solve many of the still unsolved problems, we find that there is simply a bewildering number of possibilities in reactor de-

[4] Material on atomic energy was supplemented from "Power from Atomic Energy," by Dr. Francis Simon, Clarendon Laboratory, University of Oxford, and from "The Atomic Rivals," in *The Financial Times*, August 6, 1954.

sign, each of which can produce the same technical results but with different economic advantages. The attractiveness of each and the economic evaluation of the positive and the negative features are probably impossible of assessment until actual systems have been built and operated. The solution of the main technical problems which confront us will probably not be achieved except in the process of building and operating actual reactor systems. From this standpoint, the significance of the Atomic Energy Commission's announcement of its decision to proceed with the design, construction, and operation of the 60,000 kw. so-called PWR reactor at Shippingport, Pa. is that a start has been made along the long road ahead of achieving a practical reactor capable of producing electric power for peace-time use on a competitive basis. It would appear almost inevitable that within this decade a number of the most promising types of such reactors will have been built in large experimental sizes and their lessons evaluated. A few may even have been put into commercial operation in special areas.

But again we must try to see this prospect in a clear perspective. Assuming only normal difficulties and disappointments in the developmental phase that we are now entering, the effect of atomic power on sources of energy in the United States is still bound to be almost negligible during the next ten years and only slight during the ensuing five years. In the decade after that, it may pick up to some significant figure. But a great deal of work still remains to be done before it will be possible to determine *whether*—and, more importantly, *when*—atomic power can take its position in the market place and meet the economic test of other forms of energy in areas, like the United States, having these in abundance.

The atomic energy law enacted last year will speed up research by industrial groups but few plants will be turning out power before 1959 or 1960. One expert predicts however that the new law will give a boost to the use of radioactive isotopes by industry. Already 850 to 900 companies in the United States use these isotopes in their business. Some use them to detect flaws in castings or to measure the height of liquids in containers or to measure the flow of oil through pipelines or to automatically control the thickness of steel being rolled. Firms are studying their use to kill germs and thus to preserve foods. They are already being used as catalysts in making new chemical products.

In this period of moderate downturn in business, it is comforting as well as reassuring to find some industries that continue their forward progress and give no indications that barriers to their growth are anywhere in sight. Such a one is the gas industry, which today is the sixth largest in the nation and expanding at a rate little short of phenomenal. Adding not far from a million new customers a year, it is today serving about 27 million homes, of which nearly 21 million are natural gas customers. This physical expansion has required constantly rapid en-

largement of plant facilities, with consequent heavy investment. From a $5 billion plant at the end of World War II, expenditures since then bid fair to boost aggregate investment to close to $15 billion by the end of 1955.

Some recent data supplied by the American Gas Association (AGA) give point to the optimism that characterizes industry leaders' estimates of future growth. Revenues of gas utilities from sales of gas to ultimate consumers in the year ended Dec. 31, 1953, totalled $2697 million, a rise of 10.3 per cent over 1952 sales of $2446 million. Industry accounted for the largest gain, 17.7 per cent, while residential revenues rose 7.5 per cent. Operating revenues of the gas utility and pipeline industry hit a new high in 1953 of $4020 million, a gain of 12.6 per cent over the preceding 12 months.

A number of other facts can be enumerated to show that natural gas producers have a sound basis for expecting their product to show far better than average gain in growth. Its high heating content, its ease in handling, its cleanliness and its relatively low cost have given it a popularity which makes it a formidable contender in the heating of residences. More than 12 million families now use this gas to heat their homes. This compares with 4 million such users in 1940, a 300 per cent increase. Some areas report a long waiting list for natural gas heating. It is predicted that there will be 14,500,000 homes using gas for heat in 1956.

Another facet of this growth picture is the tremendous market still far from saturated that inheres in the demand for new gas appliances. Something like a minor revolution has been developing in this field, with important improvements in gas ranges, water heaters, refrigerators, and heating units, all increasing their attraction. Beyond this, such comparatively new apparatus as air conditioning units, clothes dryers and home incinerators all add to the use of gas. A new study by the AGA on gas appliance sales potential emphasizes the size of this market. Recently it stated that if economic conditions remain at current levels until 1958, about 59 million gas appliances can be sold in the next five years. It went on to estimate that even if consumer buying power drops 10 per cent this year, or declines to an average of 20 per cent lower for the five year 1954–58 period, sales potential for these gas appliances would still exceed 37,600,000 units or above the actual five year 1948–52 sales of 36,300,000 units.

The popularity of gas with home users is equalled by the preference shown for it by industrial firms and commercial establishments. The former account for fully 55 per cent of total consumption and the latter about 15 per cent. Closeness to the gas fields is a compelling reason for a steel, aluminum, chemical or other industrial plant to use natural gas. Texas affords a prime example of the advantages to industry of operating close by tremendous reservoirs of an ideal fuel available at low cost.

But it is not only plants adjacent to gas fields that prefer gas. The advent of long-distance pipelines has given a further strong impetus to the use of this fuel, increasing its importance immensely and raising the industry to one of national scope. In the five year period 1949–53 close to $6 billion has been spent on transmission lines, about $4 billion of it on 44,000 miles of large diameter pipelines. The northeast section of the country is now getting natural gas and only one large area, the Pacific northwest, is yet to experience the benefits of this fuel. As demand rises, the price of gas is bound to rise also, narrowing the spread between it and competitive fuels. Unless new gas reserves are found fast enough to at least equal current use, the price will rise faster due to the scarcity factor. With approximately 200 trillion cubic feet of proven reserves, which may last an estimated 23 years at current rate of consumption, there is no apprehension as yet over the adequacy of the supply. But the industry warns that incentives must be such as to induce the discovery of new supplies.

A solar battery which converts the sun's radiation into useful amounts of electricity has been constructed by scientists of the Bell Telephone Laboratories. The transistor units of the new battery are made of silicon, a principal ingredient of common sand, and resemble elongated razor blades. When sunlight strikes the transistor, a particle of electricity is freed from the silicon crystal and it travels through the wires of the battery to form an electric current. One of the sun batteries powered a telephone conversation and others electrified small radio transmitters that sent out voice and music signals. There are no moving parts and nothing is destroyed or deteriorated in getting the electrons to move. Thus, the new device is expected to last for an indefinitely long period. The inventors are Gerald L. Pearson, physicist; Calvin S. Fuller, chemist, and Daryl M. Chapin, an experimental engineer. They pointed out that their invention can convert about 8 per cent of the energy of the sunlight falling on it. This means that about one square yard of battery would produce enough electricity to light a fifty-watt light bulb. Although no actual estimates of cost were given, the inventors explained that the individual units of the battery are no more difficult to manufacture than razor blades which cost about a half-cent to make in mass production. On that basis, a square yard of the solar battery would cost about five dollars. The scientists acknowledge that it would be some time before large-scale power installations would be possible. However, they expect immediate uses in communications where low power is sufficient. Attached to telephone poles the sun batteries could power a telephone system and charge storage batteries to provide power at night.

On theoretical grounds the maximum possible efficiency at which the battery can produce current is estimated at 22 per cent and the men who developed it see a good prospect of reaching 10 per cent to 15 per

cent in time. Meanwhile the present 8 per cent rating, according to Mr. Pearson, "is 10 to 15 times better than the best photo-voltaic devices available." Such devices are used in photographers' light-meters.

The recent demonstration of a solar battery is a reminder of the vast potential of this source of energy. What efforts are being made to tap solar radiation, and what are the prospects for its large-scale use?

First, a word about the potential. There's enough energy in the sun to provide man with all the heat, light and power he will need for the next two billion years. Though the earth receives but a tiny fraction of the sun's energy, this fraction adds up each year to the equivalent of 122 trillion tons of bituminous coal. In a single day, even when the sun may not appear to be overhead at high noon, just two square miles of land or sea water soak up an amount of solar energy equal to the punch of an atomic bomb. However, the quantity of heat received per unit area of the earth's surface, under the most favorable conditions, is relatively small in comparison with rates of heat transfer employed in industrial processes. At best, 2 to 3 pounds of water can be evaporated per square foot per day. If steam is to be produced and superheated to conventional temperatures for power production, less than one-half pound of steam can be generated per square foot per day, owing to the decreased fraction of the day during which the higher temperatures needed can be made available. Partly because of this factor, the only existing applications of solar energy are low-temperature space and water-heating applications which are used quite freely in the South. In such cases, water is merely heated to temperatures well below the boiling point so that heat can be absorbed and utilized or stored over many hours during clear days. When solar energy available is not sufficient in itself to meet the demands, it may be combined with conventional heating to save such fuel energy as is possible. The generation of electricity from solar radiation, through the intermediate provision of a steam or similar cycle, will undoubtedly be carried out to some degree in the future, when fossil and nuclear fuels become scarce. Costs will be high but the technical difficulties involved are not great. The real shortcoming is that this type of energy is intermittent and will not serve to supply firm power unless integrated with a hydroelectric in such a manner that storage behind the dam is allowed to gather as solar energy is fed into the network.

Even now the United States and foreign solar scientists and engineers are meeting at Phoenix, Arizona, in a symposium being held by Stanford Research Institute under sponsorship of the Association for Applied Solar Energy—a group of Southwestern industrialists, bankers, agriculturalists and educators. Some of the topics the meeting will take up: Solar engines to pump irrigation and drinking water; new food sources through better use of sunshine; heating and cooling by solar energy; design and construction of sun-operated pumps, heat collectors and stills; and electric power production from photoelectric units.

At the Southwest Research Institute on Essor Ranch, San Antonio, Texas, sun-heated water is being used to keep one of the newer buildings comfortable when cool weather hits. With simple units and no moving parts, the water can be heated to 150° Fahrenheit. The Institute's solar-energy "furnace" is serving research purposes as well as cutting the fuel bill.

Only a few years ago, Dr. William M. Conn at Rockhurst College in Kansas City, Missouri, erected a small 10-ft. parabolic aluminum mirror with which it has been possible to reach temperatures of the order of 3000° C. in 10 seconds. This furnace has been used primarily as a research unit for studying metals and heat-resistance materials—their melting points, high-temperature changes and possible uses in jet engines.

One of the largest solar furnaces in the world was built by a Frenchman, Felix Trombe, in 1952; it is a 40-foot unit in the French Pyrenees. On a reasonably sunny day it can melt about 180 pounds of iron an hour. Or, looking at it another way, it can produce about 100 kilowatts of heat-power energy. The simplest method of using solar energy, and at present one of the most efficient, is to soak up sunlight directly on a jet black surface and lead the resulting heat into a boiler. Several steam plants for generating power from sunlight in this way are now in operation.

In Florida, Texas, Arizona and California, the use of solar energy as a means of producing domestic hot water has already made headway. In these water-heaters, a network of pipes is ordinarily mounted on the roof and exposed to the sun under one or two layers of glass. The sun-heated water, at a temperature of approximately 150° Fahrenheit, is stored in an insulated tank for use as needed.

One large water-heating system in Switzerland is said to have obtained 36 per cent of the maximum heat inflow during July, with monthly averages of around 10 per cent for March and August; this is considered excellent. And during the past two years the National Physical Laboratory in Pusa, India, has been using solar energy for the production of steam for driving low-horsepower engines that operate water pumps and small looms. It has also been studying the possibilities of solar energy for household cooking. All this suggests that the day of extensive use of solar energy may be relatively close. The President's Materials Policy Commission figures that the potential market for solar heating in the U. S. may be more than 13 million installations by 1975. The report further estimates that these heating units, selling for about $2000 to $3000 each, may effectively satisfy by that time about 10 per cent of the nation's total energy requirements. There's every reason, however, to believe that commercial solar energy furnace units will appear on a large scale before that. Some methods for harnessing solar energy have been developed which are already competitive with conventional sources of power like coal, oil and gas.

Undeveloped areas in the tropics may be introduced to the sun stove for cooking purposes as a result of a $45,000 Ford Foundation grant to the research division of New York University's College of Engineering. A sun stove conceived and developed by Dr. Maria Telkes, will be tested to develop a practical stove to be mass-produced economically for average families in these areas of the world. Included in the project is an economic and sociological study in which the acceptability of the device may be determined. Dr. Telkes has designed a solar-heated house and devised a solar still that enabled shipwrecked service men in the Second World War to obtain fresh water from salt water by sunlight. She predicts that small household appliances like toasters, beaters or mixers using the sun's energy might be in fairly widespread use within the next five years.

In any case, there's little doubt the attempts to harness those billions of solar horsepower will continue at an increased tempo. Just now there is much to be said in favor of the all-electric house as eventually making available the ultimate in convenience from the point of view of our present standard of living. Cooking, cleaning, space heating and cooling, service water heating, garbage disposal, washing, refrigeration and operation of a large number of other auxiliary appliances and tools, presuppose the use of electricity. It is known that the heat rejected at thermal-generating stations to supply the public with electricity can be put to use to furnish the heat necessary to furnish space heating if the rejected heat is circulated by a warm-water loop to serve as heat source for a heat pump in each building. By reason of the higher temperature of the source, as compared to natural heat sources, the coefficient of the heat pump would be increased and its size decreased until it would be just large enough to cool the required space in summer. By this means, heating and cooling costs would be minimized and waste of energy avoided. Since warm water cannot be circulated economically over large distances, the size of the project served will be limited and an open-cycle, gas-turbine power plant with waste-heat drum may be preferred. The evolutionary trend in the active conservation of energy for space heating may force the use of much improvement in heat pumps.

The heat pump, which employs the refrigerating cycle in reverse, is not a source of energy. It does, however, conserve non-replaceable fuel for those applications where mechanical power or high temperature energy are otherwise required. The principal applications of the heat pump have been in space heating and cooling and in certain evaporative processes. In all of these applications, the heating effect obtained is several times the equivalent heat energy input to the compressor. The multiplying factor is known as the coefficient of performance. In evaporative processes, the coefficient factor of performance may be as great as twelve or more. In space heating, the chosen method of driving the

compressor is by means of an electric motor. If the assumption is made that only 25 per cent of the fuel-energy input to the steam-generating station is available at the consumer's meter, then it is certain that the heat pump must have a coefficient of at least 4 before fuel can be saved. In like manner, the efficiency of fuel utilization in space heating (including heat recovered) may be as high as 60 to 70 per cent; so that, here again, a coefficient of performance of nearly 4 is indicated as a minimum. An average of 4 has not been obtained thus far in the United States, if the heat source is the air or the ground. A coefficient of performance of 4.4 is possible with ground water at 55° F., and mechanical improvements are possible that may be able to raise the coefficient of performance slightly above 5, with a resulting saving in energy. A double-well system used at a farm or a residence would not cause a lowering of the water table and this makes it possible to predict very widespread practical use of such a scheme as this, when it is planned for both cooling as well as heating, but not for heating alone, since the extra cost of the heat pump would defeat the project economically.

We are making great strides in finding uses for what were called worthless by-products of refining. Butane and propane were once classed in this division and oil companies "flared" or "weathered" them off at refineries. But in 1953 almost 5 billion gallons of this former waste were turned into fuel—more than seven times the amount consumed in 1945. The industry currently is popularizing "LP-Gas" although it is commonly called bottled gas and sold under more than 100 commercial names. Most of the major oil producing companies have some of the business and the volume depends upon the amount of natural gas and crude petroleum being processed and how vigorously the companies have promoted this market. About one half the total production is burned for domestic and commercial purposes while the balance is used on farms, buses, fork-lift trucks and in the manufacture of butadiene rubber.

It is very difficult to predict with any degree of certainty just where the lines of endeavor for progress in energy sources will lead us. Scientists and engineers have been working for years in making available the untold energy displayed by the wind and the tides. Heat from within the earth's surface has been used in some places with excellent results. But no one has proposed a good method of placing appropriate heat-transfer devices miles below the earth's surface though it is believed that at some time in the future such a project might well be accomplished. Let us assume that the properties of the core a few miles below the surface are:

Specific Gravity	6.0
Specific Heat	0.1
Temperature	500° F.

With a heat rejection temperature of 100° F., one cubic mile of this material would deliver heat equal to that due to burning approximately 80 million tons of coal. This rough calculation does not take into account heat transmission from the surrounding mass.

Space heating, in the final analysis, comprises a problem of merely raising the surrounding temperature from a few degrees to a possible maximum of 100° F. The direct use of fuel for this purpose represents another instance of absolute waste if alternate methods can be found of using waste heat. One of the alternate means may be found in the use of modern heat pumps and the adaptation of solar energy. There is an interesting potential economic challenge in the marriage of a heat pump with solar heat sources. If the heat pump could show that it uses its same capacity for heating as it supplies for summer cooling and the remaining heating for winter is supplied by solar means, this might be an interesting combination. The alternative of providing the excess through use of waste heat is also intriguing. Acceptance of more efficient methods of heating as a means of conserving our fuel reserves or decreasing conventional energy requirements will have to take into account the provision of added benefits, such as greater convenience or the provision of some degree of air conditioning. Unless this is done, we are faced with a massive addition of power for summer cooling at the expense of energy produced in the usual manner. The great challenge we face is to devise a system that provides for a maximum of convenience, comfort, reliability at least cost expense of total energy in accordance with our rising standard of living.

Sincere acknowledgment is made to Clarence H. Linder, Vice President of the General Electric Company, New York, New York for his kindness in checking the text and making constructive suggestions without which the author would not have felt equal to the challenge of presenting this address.

632 Miles-per-Hour Sled Run.—An Air Force officer has gone from a speed of 632 miles per hour to a dead stop in less than one and a half seconds in performance of a test for the Air Research and Development Command. Lt. Colonel John P. Stapp, Chief of the Aero-Medical Field Laboratory at Holloman Air Development Center, New Mexico, reached that speed while riding a rocket-propelled sled to reproduce exposure to wind-blast and slowdown effects experienced by aircrewmen when escaping from aircraft at supersonic speeds.

The test vehicle was a 2000-lb. sled mounted on rails and pushed by nine rockets which exerted a total force of 40,000 lb. thrust. The enormous force generated by the rockets accelerated the sled from a standing start to its top speed of 632 miles per hour in five seconds time and 2800-ft. distance. After the rockets burned out, the sled coasted for less than a half second and the actual braking to a stop took a fraction more than a second. During the test the subject, Colonel Stapp, withstood a deceleration of 35 times gravity and a wind pressure of more than two tons. With the exception of a plastic helmet and a clear plastic visor to protect his face, no special clothing was worn for the test.

The sled was stripped even of a wind shield for the test. The subject was strapped to his seat with a special nylon web harness made of individual strips of nylon comprising shoulder and leg straps, chest and lap belt and tie down straps.

At the 4100-ft. altitude of Holloman Air Development Center, the wind force against Colonel Stapp's body when he reached the speed of 632 miles per hour was equivalent to that of almost 1.7 times the speed of sound (more than 1000 miles per hour) at 35,000 ft., the normal cruising altitude of jet aircraft.

Although he did not suffer any ill effect from the test, Colonel Stapp received small blood blisters from dust particles in the air and two black eyes as a result of his eyes being thrown forward against his eyelids during the rapid deceleration period. As quickly as he had been carefully examined at the Holloman Hospital after the run, he ate lunch and set about analyzing the data gained from the run.

In June of last year the Air Force announced that Lt. Col. Stapp had attained a speed of 421 miles per hour during an earlier test run which was a part of a series of tests that led to his latest speed of 632 miles per hour.

Miniature Tantalum Capacitors.—A tantalum electrolytic tubular capacitor providing unusual stability over a wide temperature range was announced recently by the Sprague Electric Company as an additional type in the company's expanded line of tubular capacitors for operation in military electronic equipment at temperatures up to 85° C.

One of a new series of Sprague's "Tantalex" line employing tantalum as the anode, the Type 102D unit is intended for low voltage applications, where small size, superior and stable electrical characteristics are required. Use of tantalum, the most stable of all anodic film-forming materials, provides exceptional stability of performance. The capacitor exhibits no shelf aging under long periods of test.

Of particular interest to the designer of military and industrial electronic equipment are the small capacitance drop-off at extremely low temperatures, the extremely low leakage current, and the low power factor of Type 102D Capacitors over an operating temperature range of −55 to +85° C.

ON THE STABILITY OF PARALLEL FLOWS WITH RESPECT TO PERIODIC DISTURBANCES

BY

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ABSTRACT

The stability of a two-dimensional parallel flow has been investigated by the energy method. It was found that the flow is stable with respect to spatially periodic disturbances of the type

$$u' = Re\,[f(y, z, t)\exp(i\lambda x)] \qquad (A)$$

where x is the distance along the direction of the basic flow. This result holds true for both finite and infinitesimal disturbances. It also holds for both two-dimensional and three-dimensional disturbances.

The analysis follows essentially the procedure used by Thomas. A comparison of the present analysis with that used by Synge is given. The reason for the different results obtained by the method of Thomas and that of Synge is pointed out.

The Poiseuille flow in a cylindrical tube of arbitrary section is also stable with respect to disturbance of the type (A).

I. INTRODUCTION

One of the most interesting problems in fluid dynamics is the problem of the stability of a laminar flow and its transition to turbulence. There are two schools of thought in regard to the cause of transition from steady to turbulent conditions. One school contends that transition is due to a definite instability of the flow, that is, to a condition in which infinitesimal disturbances grow exponentially. The second school regards the motion in most cases as definitely stable for infinitesimal disturbances, but liable to be made turbulent by suitable disturbances of finite magnitude or by sufficiently large pressure gradients.

The stability problem may be attacked by either the theory of finite disturbances or of small disturbances. Mathematical investigations of finite disturbances are mainly based on considerations of energy or of the square of the vorticity of the disturbance, because the solution of the non-linear equations satisfied by the disturbances is extremely difficult to obtain. For small disturbances, both the energy method and the vorticity method may also be used. The most extensive investigation of the stability problem seems to be treatment of parallel flows by solving the eigenvalue problem associated with the linearized equation governing the disturbance of the exponential time factor. Synge (1)[2] showed that the energy method as well as the vorticity method coalesce with the method of the exponential time-factor.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

The linearized theory of the exponential time factor predicts the existence of the so-called Tollmien-Schlichting oscillations which has been verified by experiments (2). But there are still some controversies about the application of the results of the linearized theory to the transition of laminar flow to turbulence. Theodorsen (3) showed that all two-dimensional flows of Orr-Sommerfeld type are stable while the ordinary theory of hydrodynamic stability (1) indicates that this flow is definitely unstable. Thomas (4) showed that Poiseuille motion in a tube of circular section is always stable with respect to spatially periodic disturbance of the type:

$$u' = Re[f(r,\theta) \exp(\sigma t + i\lambda x)] \tag{1}$$

where r, θ, x are the cylindrical coordinates. σ is a complex number and f is a complex function. λ is a real positive number. On the other hand, Synge showed that the same flow is stable only when the Reynolds number of the basic flow is below certain critical value. To follow up the problem, the present author tries to find out why such contradictory results are obtained. The present paper reports the results which confirm Theodorsen's result.

For simplicity, we consider the two-dimensional parallel basic flow with two-dimensional disturbances, that is, the two-dimensional flow of the Orr-Sommerfeld type, in detail in order to throw some light on the above question. It will be shown at the end of this paper that the analysis may be extended to the cases of two-dimensional parallel basic flow with three dimensional disturbances and of three-dimensional or axially symmetrical case in a straightforward manner. Stability is found in all cases.

II. FUNDAMENTAL EQUATIONS

We shall consider the steady two-dimensional flow of an incompressible viscous fluid of density ρ and viscosity μ in a region D. The velocity components and the pressure p of this basic flow are given as follows

$$u = U(y), \quad v = 0, \quad p = P(x,y) \tag{2}$$

where u and v are the x- and y- velocity components, respectively. $U(y)$ is a function of y only. We now imagine that this steady basic flow is disturbed so that the corresponding quantities in the disturbed motion are

$$u = U(y) + u'(x,y,t), \quad v = v'(x,y,t), \quad p = P(x,y) + p'(x,y,t) \tag{3}$$

where prime refers to the perturbed quantities. We assume that u', v', and p' are suitably restricted as regards continuity and differentiability. Both the basic flow and the disturbed flow satisfy the Navier-Stokes equations and the equation of continuity. Thus we have

$$\frac{\partial u'}{\partial t} + (U + u')\frac{\partial u'}{\partial x} + v'\frac{dU}{dy} + v'\frac{\partial u'}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{1}{\rho}\frac{\partial p'}{\partial x} = \delta\left(\nabla^2 u' + \frac{d^2U}{dy^2}\right) \quad (4)$$

$$\frac{\partial v'}{\partial t} + (U + u')\frac{\partial v'}{\partial x} + v'\frac{\partial v'}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{1}{\rho}\frac{\partial p'}{\partial y} = \delta\nabla^2 v' \quad (5)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 \quad (6)$$

$$\frac{1}{\rho}\frac{\partial P}{\partial x} = \delta\frac{d^2U}{dy^2} \quad (7)$$

$$\frac{1}{\rho}\frac{\partial P}{\partial y} = 0 \quad (8)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \text{ and } \delta = \mu/\rho'.$$

From Eqs. 4 and 7, and 5 and 8 we have, respectively,

$$\frac{\partial u'}{\partial t} + (U + u')\frac{\partial u'}{\partial x} + v'\frac{dU}{dy} + v'\frac{\partial u'}{\partial y} + \frac{1}{\rho}\frac{\partial p'}{\partial x} = \delta\nabla^2 u' \quad (9)$$

$$\frac{\partial v'}{\partial t} + (U + u')\frac{\partial v'}{\partial x} + v'\frac{\partial v'}{\partial y} + \frac{1}{\rho}\frac{\partial p'}{\partial y} = \delta\nabla^2 v'. \quad (10)$$

Equations 6, 9 and 10 are the disturbance equations which hold for both finite disturbances and infinitesmal disturbances. For infinitesimal disturbances, it is customary to neglect the second order terms such as $u'\frac{\partial u'}{\partial x}$, etc., then Eqs. 9 and 10 become, respectively,

$$\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} + v'\frac{dU}{dy} + \frac{1}{\rho}\frac{\partial p'}{\partial x} = \delta\nabla^2 u' \quad (9a)$$

$$\frac{\partial v'}{\partial t} + U\frac{\partial v'}{\partial x} + \frac{1}{\rho}\frac{\partial p'}{\partial y} = \delta\nabla^2 v'. \quad (10a)$$

Equations 6, 9*a* and 10*a* are the well known linearized disturbance equations which have been extensively studied. The well known Orr-Sommerfeld equation is obtained from these equations by assuming x-wise spatially periodic disturbance with exponential time factor.

III. THE ENERGY EQUATION

We are going to find the rate of change of the kinetic energy of the disturbance in the domain D. If the kinetic energy of the disturbance decreases with increase of time, the original flow is stable. Before we derive the expression of the rate of change of kinetic energy of the disturbance in the domain D, we have to define the domain D and to say something about the nature of the disturbances on the boundary of the domain D. Since we study the problem of two-dimensional parallel flows, it is convenient to take D as a rectangle bounded on two sides by lines $x = x_0$ and $x = x_1$ and on the other two sides by the lines $y = y_0$ and $y = y_1$. We assume that on the lines $y = y_0$ and $y = y_1$, $u' = v' = p' = 0$. These lines may be considered as the solid walls or as lines situated in the fluid outside the region of disturbed flow.

On the lines $x = x_0$ and $x = x_1$, two different assumptions may be made. One is that the disturbances are periodic with respect to x so that the values of u' and $v'$and p' are identical at both boundaries, that is, u', v', and p' have the same values at corresponding points in the two lines $x = x_0$ and $x = x_1$. The other assumption is that the lines $x = x_0$ and $x = x_1$ are situated outside the region of the disturbed flow so that at these lines $u' = v' = p' = 0$. It will be shown later that both assumptions will give the same result of the energy equation 16.

Now we multiply (9) by $\rho u'$, (10) by $\rho v'$, and add the resultant equations. We have then

$$\frac{\partial E'}{\partial t} + (U + u')\frac{\partial E'}{\partial x} + v'\frac{\partial E'}{\partial y} + \rho u'v'\frac{dU}{dy} + u'\frac{\partial p'}{\partial x} + v'\frac{\partial p'}{\partial y} = \mu(u'\nabla^2 u' + v'\nabla^2 v') \quad (11)$$

where

$$E' = \frac{\rho}{2}(u'^2 + v'^2).$$

On applying the equation of continuity (6), we have

$$\frac{\partial E'}{\partial t} + \frac{\partial (U + u')E'}{\partial x} + \frac{\partial v'E'}{\partial y} + \rho u'v'\frac{dU}{dy} + \frac{\partial p'u'}{\partial x} + \frac{\partial p'v'}{\partial y} = \mu(u'\nabla^2 u' + v'\nabla^2 v'). \quad (12)$$

We integrate Eq. 12 throughout the domain D, and have

$$\frac{\partial E}{\partial t} = -\rho \iint_D u'v'\frac{dU}{dy}\,dx\,dy + \mu \iint_D (u'\nabla^2 u' + v'\nabla^2 v')dx\,dy \quad (13)$$

where

$$E = \frac{\rho}{2}\iint_D (u'^2 + v'^2)dx\,dy.$$

The integrals of all the terms other than the three terms in (13) are zero because of the boundary conditions. For instance,

$$\iint_D \frac{\partial (U+u')E'}{\partial x}\,dx\,dy = \int_{y_0}^{y_1} \{[(U+u')E']_{x=x_1} - [(U+u')E']_{x=x_0}\}dy = 0. \quad (14)$$

It should be noted that the same energy equation (13) will be obtained if we start from the linearized equation (9*a*) and (10*a*) instead of (9) and (10), because the non-linear terms $u'\dfrac{\partial u'}{\partial x}$ etc. do not contribute anything in (13) as shown in (14). However, Eq. 9 will give an important relation (19) to prove the stability of the flow that Eq. 9*a* cannot give.

The last integral of Eq. 13 may be transformed into the following form

$$\begin{aligned}\iint_D (u'\nabla^2 u' + v'\nabla^2 v')dx\,dy &= \iint_D \left\{ \frac{\partial}{\partial x}\left(u'\frac{\partial u'}{\partial x}\right) - \left(\frac{\partial u'}{\partial x}\right)^2 \right. \\ &+ \frac{\partial}{\partial y}\left(u'\frac{\partial u'}{\partial y}\right) - \left(\frac{\partial u'}{\partial y}\right)^2 + \frac{\partial}{\partial x}\left(v'\frac{\partial v'}{\partial x}\right) - \left(\frac{\partial v'}{\partial x}\right)^2 \\ &\left. + \frac{\partial}{\partial y}\left(v'\frac{\partial v'}{\partial y}\right) - \left(\frac{\partial v'}{\partial y}\right)^2 \right\} dx\,dy \\ &= -\iint_D \left\{ \left(\frac{\partial u'}{\partial x}\right)^2 + \left(\frac{\partial u'}{\partial y}\right)^2 + \left(\frac{\partial v'}{\partial x}\right)^2 + \left(\frac{\partial v'}{\partial y}\right)^2 \right\} dx\,dy. \quad (15)\end{aligned}$$

The energy equation (13) becomes

$$\begin{aligned}\frac{\partial E}{\partial t} &= -\rho \iint_D u'v'\frac{dU}{dy}\,dx\,dy - \mu \iint_D \left\{ \left(\frac{\partial u'}{\partial x}\right)^2 + \left(\frac{\partial u'}{\partial y}\right)^2 \right. \\ &\left. + \left(\frac{\partial v'}{\partial x}\right)^2 + \left(\frac{\partial v'}{\partial y}\right)^2 \right\} dx\,dy = -\rho M - \mu N \quad \text{(say)}. \quad (16)\end{aligned}$$

Equation 16 is the standard energy equation used in the investigation of stability of laminar flow. Synge (1) used this equation to find the sufficient condition for stability of the parallel flow.

Thomas (4) tried to find another relation for the first integral on the right hand side of (16) in essentially the following manner:

Multiplying Eq. 9 by $\rho U(y)$ of the basic flow and integrating the resulting equation, throughout the domain D, we have

$$\rho \iint_D U \frac{\partial u'}{\partial t}\, dx\, dy = \rho \frac{\partial}{\partial t} \iint_D U u'\, dx\, dy \tag{17a}$$

$$\rho \iint_D U \left(u' \frac{\partial u'}{\partial x} + v' \frac{\partial u'}{\partial y} \right) dx\, dy = \rho \iint_D U \times \left(\frac{\partial u'^2}{\partial x} + \frac{\partial u'v'}{\partial y} \right) dx\, dy = -\rho \iint_D u'v' \frac{dU}{dy}\, dx\, dy \tag{17b}$$

$$\mu \iint_D U \frac{\partial^2 u'}{\partial y^2}\, dx\, dy = -\mu \iint_D \frac{\partial u'}{\partial y} \frac{dU}{dy}\, dx\, dy = -\mu \iint_D \left\{ \frac{\partial}{\partial y} \left(u' \frac{dU}{dy} \right) - u' \frac{d^2U}{dy^2} \right\} dx\, dy = \mu \iint u' \frac{d^2U}{dy^2}\, dx\, dy. \tag{17c}$$

The integrals of all the terms in this resultant equation other than the above three are zero because of the boundary conditions and the equation of continuity. For instance

$$\rho \iint_D Uv' \frac{dU}{dy}\, dx\, dy = \rho \int_{x_0}^{x_1} \left\{ \left(\frac{v'U^2}{2} \right)_{y=y_1} - \left(\frac{v'U^2}{2} \right)_{y=y_0} \right\} dy - \frac{\rho}{2} \iint_D U^2 \frac{\partial v'}{\partial y}\, dx\, dy = -\frac{\rho}{2} \iint_D U^2 \frac{\partial u'}{\partial x}\, dx\, dy = 0. \tag{18}$$

From Eq. 17 we have

$$\rho M = \rho \iint_D u'v' \frac{dU}{dy}\, dx\, dy = \rho \frac{\partial}{\partial t} \iint_D U u' dx\, dy - \mu \iint_D u' \frac{d^2U}{dy^2}\, dx\, dy. \tag{19}$$

The integral on the left hand side ρM is known as the amount of work produced by the basic flow and contributed to the disturbing motion. Hence ρM is the only motive in the energy equation (16) which causes the possible instability of the flow. It is interesting to notice that ρM

may be expressed in terms of integrals of u' and some functions of basic flow only. It is this relation which shows that ρM, the cause of instability, is zero for spatially periodic disturbance (see next section).

Another important point which should be mentioned is that Eq. 19 cannot be obtained from the linearized Orr-Sommerfeld equation (9*a*), because the important terms in (17*b*) are neglected there. From (9*a*) we will have

$$\rho \frac{\partial}{\partial t} \iint_D Uu' dx\, dy - \mu \iint_D u' \frac{d^2U}{dy^2} dx\, dy = 0. \tag{20}$$

Hence we miss the information given by (19), namely that M is determined by u' and U. If we use the relation of (19) and assume that the disturbance is spatially periodic, we shall show that M vanishes. The energy equation (16) becomes

$$\frac{\partial E}{\partial t} = -\mu \iint_D \left\{ \left(\frac{\partial u'}{\partial x}\right)^2 + \left(\frac{\partial u'}{\partial y}\right)^2 + \left(\frac{\partial v'}{\partial x}\right)^2 + \left(\frac{\partial v'}{\partial y}\right)^2 \right\} dx\, dy. \tag{21}$$

From (21), we shall show that the flow is stable.

IV. SPATIALLY PERIODIC DISTURBANCES

If the disturbance is spatially periodic along the x-direction, it may be expressed in the following manner

$$\psi' = f(y,t) \exp(i\lambda x) + f_c(y,t) \exp(-i\lambda x) \tag{22}$$

where ψ' is the stream function for the disturbance such that

$$\frac{\partial \psi'}{\partial y} = u', \quad \frac{\partial \psi'}{\partial x} = -v'. \tag{23}$$

λ is the wave number which is a positive real number. $f(y,t)$ is a complex function of y and t. f_c is the complex conjugated function of f.

From Eqs. 22 and 23, we have

$$u' = f' \exp(i\lambda x) + f_c' \exp(-i\lambda x) \tag{24}$$

where

$$f' = \frac{\partial f}{\partial y} \quad \text{and} \quad f_c' = \frac{\partial f_c}{\partial y}.$$

Substituting (24) into (19), we have

$$\rho M = \rho \iint_D u'v' \frac{dU}{dy} dx\, dy = 0. \tag{25}$$

Hence for the disturbance of the type of (22), the energy equation (16) is reduced to Eq. 21. Substituting Eqs. 22 and 23 into Eq. 21, we have

$$\frac{\partial E}{\partial t} = -\mu \left\{ 2 \iint_D (f'' f_c'' + 2\lambda^2 f' f_c' + \lambda^4 f f_c) \, dx \, dy \right.$$

$$+ \iint_D [(f''^2 - 2\lambda^2 f'^2 + \lambda^4 f^2) \exp(2i\lambda x)] \, dx \, dy$$

$$\left. + \iint_D [(f_c''^2 - 2\lambda^2 f_c'^2 + \lambda^4 f_c^2) \exp(-2i\lambda x)] \, dx \, dy \right.$$

$$= -2\mu(I_2^2 + 2\lambda^2 I_1^2 + \lambda^4 I_0^2) \tag{26}$$

where

$$f'' = \frac{\partial^2 f}{\partial y^2} \quad \text{and} \quad f_c'' = \frac{\partial^2 f_c}{\partial y^2}$$

$$I_0^2 = \iint_D f f_c \, dx \, dy, \quad I_1^2 = \iint_D f' f_c' \, dx \, dy,$$

$$I_2^2 = \iint_D f'' f_c'' \, dx \, dy. \tag{27}$$

I_0^2, I_1^2 and I_2^2 are positive numbers for any disturbances. The second and third integrals on the right hand of Eq. 26 are zero because of the periodicity in x. Hence $\frac{\partial E}{\partial t}$ is always negative. The flow is stable with respect to the disturbance of the type of (22).

It should be noted that if we do not use the relation (19) and substitute (22) and (23) into the energy equation (16), we have

$$\frac{\partial E}{\partial t} = i\lambda\rho \iint_D (f_c' f - f' f_c) \frac{dU}{dy} \, dx \, dy - 2\mu(I_2^2 + 2\lambda^2 I_1^2 + \lambda^4 I_0^2). \tag{28}$$

In Synge's notation (1), Eq. 28 becomes

$$\frac{\partial E}{\partial t} = i\lambda\rho Q - 2\mu(I_2^2 + 2\lambda^2 I_1^2 + \lambda^4 I_0^2). \tag{29}$$

From Eq. 29, Synge gives the sufficient conditions for stability of the flow. His results do not tell us anything if these sufficient conditions are violated. From the present more accurate analysis, by Eq. 19 we show that $Q = 0$ for the disturbance of the form of (22). We thus show that even if Synge's sufficient conditions for stability of parallel flow are

violated, the flow is still stable. In fact, all the two-dimensional parallel flows of Eq. 2 are stable with respect to disturbances of the type of (22).

The disturbances (22) are a special case of disturbances represented by

$$\psi' = Re[f_1(y,t)\exp(i\lambda_1 x) + f_2(y,t)\exp(i\lambda_2 x) + \ldots]. \tag{30}$$

It is easy to show that if $\lambda_j \neq 0$, the ρM given by (19) for the disturbances of (30) will be zero. The flow is then stable. The solutions predicted by the Orr-Sommerfeld equation are of the type of (30). From the present analysis, it seems to indicate that the eigen oscillations ($\lambda_j \neq 0$) will not grow up into disturbances of finite amplitude, to produce transition, because such a disturbance cannot take energy from the mean flow due to the non-linear relation (19).

V. EXTENSION OF THE ABOVE ANALYSIS

The present analysis may be easily extended to the following two cases:

(*a*) Two dimensional parallel basic flow with three dimensional disturbances. The basic flow is

$$u = U(y), \quad v = w = 0, \quad p = P(x,y,z). \tag{31}$$

The disturbed motion is

$$\begin{aligned} u &= U(y) + u'(x,y,z,t), \quad v = v'(x,y,z,t) \\ w &= w'(x,y,z,t), \quad p = P(x,y,z) + p'(x,y,z,t). \end{aligned} \tag{32}$$

Now we take the three-dimensional domain D_3 as a box bounded by planes $x = x_0$, $x = x_1$, $y = y_0$, $y = y_1$, $z = z_0$ and $z = z_1$. On the planes $y = y_0$ and $y = y_1$, we assume that $u' = v' = w' = p' = 0$, while on planes $x = x_0$, $x = x_1$ and $z = z_0$, $z = z_1$, we may assume either that the disturbances are periodic in x and z so that the disturbances at corresponding points on $x = x_0$, $x = x_1$, (or $z = z_0$, $z = z_1$) are the same, or that disturbances vanish on $x = x_0$, $x = x_1$ (or $z = z_0$, $z = z_1$). The energy equation is then

$$\begin{aligned} \frac{\partial E_3}{\partial t} = &- \rho \iiint_{D_3} u'v' \frac{dU}{dy}\, dx\, dy\, dz - \mu \iiint_{D_3} \left\{ \left(\frac{\partial u'}{\partial x}\right)^2 \right. \\ &+ \left(\frac{\partial u'}{\partial y}\right)^2 + \left(\frac{\partial u'}{\partial z}\right)^2 + \left(\frac{\partial v'}{\partial x}\right)^2 + \left(\frac{\partial v'}{\partial y}\right)^2 + \left(\frac{\partial v'}{\partial z}\right)^2 \\ &\left. + \left(\frac{\partial w'}{\partial x}\right)^2 + \left(\frac{\partial w'}{\partial y}\right)^2 + \left(\frac{\partial w'}{\partial z}\right)^2 \right\} dx\, dy\, dz = - \rho M_3 - \mu N_3 \end{aligned} \tag{33}$$

where

$$E_3 = \frac{\rho}{2}\iiint_{D_3} (u'^2 + v'^2 + w'^2)dx\,dy\,dz.$$

For the axial (x-wise) equation of motion of the disturbance, we also have the relation

$$\rho M_3 = \rho \iiint_{D_3} u'v' \frac{dU}{dy}\,dx\,dy\,dz = \rho \frac{\partial}{\partial t}\iiint_{D_3} Uu'dx\,dy\,dz - \mu \iiint_{D_3} u' \frac{d^2U}{dy^2}\,dx\,dy\,dz. \quad (34)$$

For periodic disturbance of the type

$$u' = Re[f(y,z,t) \exp(i\lambda x)] \quad (35)$$

etc., it is easily shown from (34) that

$$\rho M_3 = \rho \iiint_{D_3} u'v' \frac{dU}{dy}\,dx\,dy\,dz = 0. \quad (36)$$

Hence Eq. 33 becomes

$$\frac{\partial E_3}{\partial t} = -\mu N_3. \quad (37)$$

Since N_3 is a positive number, $\frac{\partial E_3}{\partial t}$ is always negative; thus the flow is stable with respect to periodic disturbance of the type of (34).

(*b*) Poiseuille motion in a cylindrical tube of arbitrary section. The basic flow is

$$u = U(y,z), \quad v = w = 0, \quad p = P(x,y,z) \quad (38)$$

where x is the axial direction. The disturbed motion is

$$u = U(y,z) + u'(x,y,z,t), \quad v = v'(x,y,z,t)$$
$$w = w'(x,y,z,t), \quad p = P(x,y,z) + p'(x,y,z,t). \quad (39)$$

Now we take the three-dimensional domain D_P as a section of the tube bounded by two planes $x = x_0$ and $x = x_1$. The disturbances u', v', w', and p' are zero on the wall of the tube. On the planes $x = x_0$ and $x = x_1$, we may assume that the disturbances vanish or that the disturbances are periodic in x so that at corresponding pints on $x = x_0$ and $x = x_1$, the values of u', v', w' and p' are the same. The energy equation

for such a domain D_P is then

$$\frac{\partial E_P}{\partial t} = -\rho \iiint_{D_P} \left(u'v' \frac{\partial U}{\partial y} + u'w' \frac{\partial U}{\partial z} \right) dx\, dy\, dz$$

$$- \mu \iiint_{D_P} \left| \left(\frac{\partial u'}{\partial x} \right)^2 + \left(\frac{\partial u'}{\partial y} \right)^2 + \left(\frac{\partial u'}{\partial z} \right)^2 + \left(\frac{\partial v'}{\partial x} \right)^2 \right.$$

$$+ \left(\frac{\partial v'}{\partial y} \right)^2 + \left(\frac{\partial v'}{\partial z} \right)^2 + \left(\frac{\partial w'}{\partial x} \right)^2 + \left(\frac{\partial w'}{\partial y} \right)^2$$

$$\left. + \left(\frac{\partial w'}{\partial z} \right)^2 \right| dx\, dy\, dz = -\rho M_P - \mu N_P \quad (40)$$

where

$$E_P = \frac{\rho}{2} \iiint_{D_P} (u'^2 + v'^2 + w'^2) dx\, dy\, dz.$$

From the x-wise equation of motion of the disturbance, we have the relation

$$\rho M_P = \rho \iiint_{D_P} \left(u'v' \frac{\partial U}{\partial y} + u'w' \frac{\partial U}{\partial z} \right) dx\, dy\, dz$$

$$= \rho \frac{\partial}{\partial t} \iiint_{D_P} U u'\, dx\, dy\, dz$$

$$- \mu \iiint_{D_P} u' \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) dx\, dy\, dz. \quad (41)$$

For periodic disturbance of the type

$$u' = Re[f(y,z,t) \exp(i\lambda x)] \quad (42)$$

it is easily shown from (41) that

$$\rho M_P = \rho \iiint_{D_P} \left(u'v' \frac{\partial U}{\partial y} + u'w' \frac{\partial U}{\partial z} \right) dx\, dy\, dz = 0. \quad (43)$$

Hence the energy equation (40) becomes

$$\frac{\partial E_P}{\partial t} = -\mu N_P. \quad (44)$$

Since N_P is a positive number for any disturbance $\frac{\partial E_P}{\partial t}$ is always nega-

tive; thus the flow is always stable with respect to periodic disturbances of the type of (41).

A special case of this problem is the Poiseuille flow in a straight circular pipe which was investigated by both Synge (1) and Thomas (4). Thomas showed that the flow is stable with respect to disturbances which are periodic with respect to the axial coordinates (42) or (1). The important discovery of Thomas is the relation (41) in cylindrical coordinates. Synge did not use the relation (41) to show that $\rho M_P = 0$ for spatially periodic disturbances. In Synge's analysis, he assumed that $\rho M_P \neq 0$, for spatially periodic disturbances of (42); hence in his analysis, all disturbances including those which do not satisfy the equation of motion are allowed. Thus Synge could show only that when the Reynolds number is below a certain critical value, the flow is stable, instead of the complete result that the flow is always stable with respect to the spatially periodic disturbances.

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ON ASYNCHRONOUS ACTION *

BY

N. MINORSKY [1]

ABSTRACT

This paper investigates the non-linear phenomena of the asynchronous action which manifest themselves in two different ways: the asynchronous quenching and the asynchronous excitation. It is shown that the former phenomenon is due to the asymptotic loss of stability and the latter results from the change of the topological configuration of integral curves.

1. INTRODUCTION

In this paper we shall investigate the phenomena of the so called *asynchronous action* which manifests itself in two different ways—the *asynchronous quenching* and the *asynchronous excitation*.

It is frequently observed that an external periodic excitation with frequency ω not standing in any rational ratio to the frequency of the existing oscillation (which in the following will be assumed to be $\omega = 1$) extinguishes or "quenches" the latter. In some other cases, on the contrary, a potentially possible but not yet existing oscillation is brought into play or is "excited" by providing such asynchronous frequency. The term "asynchronous" merely emphasizes the lack of any rational ratio between these two frequencies—the self-excited and the externally applied.

It is obvious that these phenomena differ essentially from the phenomena of non-linear or subharmonic resonance in which the existence of a rational ratio between two frequencies is always essential.

As far as known these effects were observed for the first time by Groszkowski (1)[2] but their theory was developed later by Russian physicists L. Mandelstam and N. Papalexi (2) and, still later by Kobsareff (3) and some others.

The theory of the Russian authors makes use of the so-called "equivalent parameters" introduced in the theory of equivalent linearization and, in fact, is a further extension of this theory when there are present two frequencies instead of one.

It seemed interesting, however, to trace out these effects directly from the differential equations without making use of a somewhat artificial tool of the equivalent linearization. This formulates the principal purpose of this paper.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Although, from an offhand consideration, the "quenching" and the "excitation" seem to appear as inverse effects, in reality their nature is quite different. In fact, the first effect is observed generally in "soft" systems, whereas the latter exists only in "hard" ones. We use the terms "soft" and "hard" in the usually employed sense to designate the well known properties of non-linear characteristics. As these two characteristics correspond to different topological configurations of singularities and limit cycles, the two phenomena are hardly comparable.

In the first five sections we shall outline a general method of approach to these problems after which the analysis of asynchronous quenching (Section 5) and of asynchronous excitation (Section 6) will be a relatively simple matter.

2. FORMULATION OF THE PROBLEM

In what follows we shall be concerned with a differential equation of the form

$$\ddot{x} - (a - cx^2)\dot{x} + x = \gamma \sin \omega t \tag{1}$$

which is a van der Pol equation with a periodic right hand term. In order to simplify the problem, we shall assume that the parameters, a, c and γ are small numbers of the same order of magnitude. Our primary object will be to investigate the existence and stability of the periodic solutions of Eq. 1 when ω varies. Occasionally we shall investigate also the effect of the parameter γ, but this is of a rather secondary importance in this problem. The advantage of the assumption regarding the smallness of parameters is in that it permits obtaining some conclusions already from the first approximation otherwise the problem becomes more complicated.

If $\gamma = 0$ one has the well known case of the van der Pol equation characterized by the existence of a stable stationary amplitude and an arbitrary phase. If $\gamma \neq 0$ the amplitude changes somewhat but still remains stable and the phase, instead of being arbitrary, is now determined from the differential equation.

In addition to the questions of amplitude and phase, there appears also the question of synchronization or "entrainment of frequency." Inasmuch as the frequency of the left hand term of Eq. 1 is $\omega = 1$ and we wish to investigate the behavior of solutions for a wide interval of the frequency ω, the synchronization effect appears here only as a *local effect* confined to a relatively narrow band around the frequency $\omega = 1$. For that reason we shall not be concerned with this matter here especially because our interest will lie principally in the asymptotic range of frequencies where the question of synchronization does not appear anyhow.

In the problem of asynchronous excitation, instead of Eq. 1 it is necessary to use the differential equation of the "hard" excitation in the form

$$\ddot{x} - (a + cx^2 - ex^4)\dot{x} + x = \gamma \sin \omega t \tag{2}$$

corresponding to a "hard" characteristic. In this connection there appears a certain difficulty as will be explained in Section 6.

3. STROBOSCOPIC TRANSFORMATION

We begin by representing Eq.1 as a system of two differential equations of the first order, *viz.*

$$\dot{x} = y; \quad \dot{y} = ay - cx^2y - x + \gamma \sin \omega t \tag{3}$$

in which we introduce the new variables $r = \sqrt{x^2 + y^2}$; $\psi = \arctan(y/x)$. This results in the system of two differential equations of the first order

$$dr/dt = ar \sin^2\psi - cr^3 \cos^2\psi \sin^2\psi + \gamma \sin\beta \sin\omega t \tag{4}$$

$$d\psi/dt = -1 + a \sin\psi \cos\psi - cr^2 \cos^3\psi \sin\psi + (\gamma/r) \cos\psi \sin\omega t. \tag{5}$$

If one tries to satisfy this system by series solutions of the form

$$r(t) = \sum_{s=0}^{\infty} \epsilon^s r_s(t); \quad \psi(t) = \sum_{s=0}^{\infty} \epsilon^s \psi_s(t)$$

where ϵ is a small parameter, one sees at once that the zero order terms in these series solutions are

$$r_0(t) = r_0; \quad \psi_0(t) = \phi_0 - t \tag{6}$$

where r_0 and ϕ_0 are arbitrary constants. One replaces these zero order terms in (4) and (5) and obtains the following differential equation representing the first order corrections:

$$dr_1/dt = Ar_0 \sin^2\psi_0 - Cr_0^3 \cos^2\psi_0 \sin^2\psi_0 + \Gamma \sin\psi_0 \sin\omega t \tag{7}$$

$$d\psi_1/dt = A \sin\psi_0 \cos\psi_0 - Cr_0^2 \cos^2\psi_0 \sin\psi_0 + \Gamma \cos\psi_0 \sin\omega t \tag{8}$$

where $A = a/\epsilon$; $C = c/\epsilon$ and $\Gamma = \gamma/\epsilon$.

We apply now the stroboscopic transformation which consists in considering the difference equations obtained by introducing the differences $r_1(2\pi) - r_1(0)$ and $\psi_1(2\pi) - \psi_1(0)$ of the functions $r_1(t)$ and $\psi_1(t)$ during one interval 2π. We define $r_1(0) = 0$ and $\psi_1(0) = 0$ so that the differences are obtained by integrating the right hand sides of (7) and (8) between 0 and 2π. This gives

$$r_1(2\pi) = [(\tfrac{1}{2}) Ar_0 - (\tfrac{1}{8}) Cr_0^3 + (\Gamma/2\pi) \int_0^{2\pi} \sin(\phi_0 - t) \sin\omega t \, dt] 2\pi \tag{9}$$

$$\psi_1(2\pi) = [\Gamma/2\pi r_0) \int_0^{2\pi} \cos(\phi_0 - t) \sin\omega t \, dt] 2\pi. \tag{10}$$

The remaining periodic terms disappear in the integration between 0 and 2π.

As to the functions $r(2\pi)$ and $\psi(2\pi)$ themselves, their differences are $r(2\pi) = [\ ]2\pi\epsilon$; $\psi(2\pi) = [\ ]2\pi\epsilon$, where the symbols $[\ ]$ correspond to the quantities in square brackets in (9) and (10).

If one defines $2\pi\epsilon = \Delta\tau$, the preceding expressions are $\Delta r/\Delta\tau = [\ ]$; $\Delta\psi/\Delta\tau = [\ ]$ where $\Delta r = r(2\pi)$ and $\Delta\psi = \Delta\phi = \psi(2\pi)$.

At the limit one gets the "stroboscopic differential equations," after the integrals in the preceding expressions are calculated. The values of these integrals are

$$\int_0^{2\pi} \sin(\phi_0 - t)\sin\omega t\, dt = -(1/(\omega^2 - 1))[\sin 2\pi\omega\cos\phi_0 + \omega(\cos 2\pi\omega - 1)\sin\phi_0] = -(1/(\omega^2 - 1))\, Q$$

$$\int_0^{2\pi} \cos(\phi_0 - t)\sin\omega t\, dt = (1/(\omega^2 - 1))[\sin 2\pi\omega\sin\phi_0 - \omega(\cos 2\pi\omega - 1)\cos\phi_0] = (1/(\omega^2 - 1))R$$

$$Q = m\cos\phi_0 + \omega n\sin\phi_0; \quad R = m\sin\phi_0 - \omega n\cos\phi_0; \quad m = m(\omega) = \sin 2\pi\omega \quad \text{and} \quad n = n(\omega) = \cos 2\pi\omega - 1 = -2\sin^2\pi\omega.$$

With these values of the integrals, the stroboscopic system is

$$dr/d\tau = -\sigma(r^3 - pr + q); \quad d\phi/d\tau = gR/r \tag{11}$$

where $\sigma = C/8$; $p = 4A/C$; $q = q(\omega) = hQ$; $h = h(\omega) = 4\gamma/\pi c \times (\omega^2 - 1)$; $g = g(\omega) = \gamma/2\pi c(\omega^2 - 1)$.

The singular point of the system (11) is given by the expressions

$$f(r_0) = r_0^3 - pr_0 + q = 0 \tag{12}$$

$$R(\phi_0) = m\sin\phi_0 - \omega n\cos\phi_0 = 0. \tag{13}$$

The stationary phase is determined from (13), *viz.*:

$$\tan\phi_0 = \omega n/m = -\omega\tan\pi\omega. \tag{14}$$

Since to a given value of $\tan\phi_0$ correspond two values of the argument, both ϕ_0 and $\phi_0 + \pi$ satisfy (14). Of these two values, one gives a stable phase and the other the unstable.

4. STABILITY OF THE STATIONARY STATE

In forming the variational equations one ascertains the following relations

$$Q(\phi_0 + \delta\phi) = Q(\phi_0) - R(\phi_0)\delta\phi = Q_0 - R_0\delta\phi;$$
$$R(\phi_0 + \delta\phi) = R_0 + Q_0\delta\phi.$$

With these notations the variational equations are

$$d\delta r/d\tau = [-\sigma(3r_0^2 - p)]\delta r + [h\sigma R_0]\delta\phi$$
$$d\delta\phi/d\tau = [-gR_0/r_0^2]\delta r + [gQ/r_0]\delta\phi$$

but, since $R_0 = 0$, these equations become

$$d\delta r/d\tau = -\sigma(3r_0^2 - p)\delta r \tag{15}$$

$$d\delta\phi/d\tau = (gQ_0/r_0)\delta\phi. \tag{16}$$

From (15) it follows that the stationary amplitude is stable if

$$r_0 > \sqrt{p/3} \tag{17}$$

and the phase is stable if the function gQ is negative. Since (14) gives two phases ϕ_0 and $\phi_0 + \pi$ for a given value of $\omega n/m$, the phase will settle at such value for which gQ_0 is negative. If one defines an essentially positive function $s(\omega) = |gQ_0|$, Eq. 16 is

$$d\delta\phi/d\tau = -s(\omega)\delta\phi. \tag{18}$$

From the expressions for g and Q it is observed that for stability of the phase $\omega^2 - 1$ and Q must be of opposite signs and, as we shall be interested here in the case when $\omega \gg 1$, Q must be negative which merely determines the stable phase. The same argument applies to the function $q(\omega) = hQ$ and results in the introduction of another positive function $v(\omega) = |hQ|$.

The variational equations (15) and (16) result in the characteristic equation

$$S^2 + [\sigma(3r_0^2 - p) + s(\omega)]S + \sigma(3r_0^2 - p)\, s(\omega) = 0. \tag{19}$$

It is easy to show now that the condition (17) is always fulfilled in view of the well known property of the roots of a cubic equation. In fact, according to the sign of the discriminant $\Delta = q^2/4 - p^3/27$ (it is to be noted that p of the general theory is negative here so that we write it as $-p$, $p > 0$) there are two cases to be considered, *viz.*:

If $\Delta > 0$, there is only one real root and, as in this case $q = -v$, $v > 0$, this root is positive and is, therefore, adequate to represent the stationary amplitude r_0. This root is given by the formula $r_0 = 2\sqrt{p/3}$ cosh $(u/3)$, where u is obtained from the expression cosh $u = (v/2) \times (3/p)^{3/2}$ so that the condition (17) reduces here to the inequality

$$\cosh(u/3) > \tfrac{1}{2} \tag{20}$$

which is always fulfilled since cosh $x \geqslant 1$ for any x.

If $\Delta < 0$, (12) has three real roots given by the formulas $r_0' = -k \times \cos(u/3)$; $r_0'' = k\cos(\pi + u)/3$; $r_0''' = k\cos(\pi - u)/3$ where $k = 2 \times \sqrt{p/3}$ and u is given by $\cos u = (v/2)(3/p)^{3/2}$.

The root r_0' is to be rejected as being negative; as to positive roots r_0'' and r_0''', it is noted that, as u varies in the interval $(0, \pi/2)$, $r_0'' < \sqrt{p/3}$ and $r_0''' > \sqrt{p/3}$ so that the first root is unstable and the second is stable. Moreover, the root r_0'' has no physical significance in this case as it does not fit into the topological configuration since the singular point is also unstable. It is to be rejected, therefore, for the same reason as r_0'.

Summing up, in the interval in which $\Delta > 0$ there is only one root r_0 capable of representing the stationary amplitude and in the interval $\Delta < 0$, there is also one root r_0''' and both these roots are stable.

5. ASYMPTOTIC INSTABILITY; ASYNCHRONOUS QUENCHING

So far we were unable to account for the phenomenon of asynchronous quenching inasmuch as the preceding analysis shows that both amplitude and phase remain stable for all values of ω. Under this condition one does not see at all why the oscillation should vanish as is observed.

This difficulty is due to the *asymptotic instability* which escapes detection on the basis of the usual definition of stability with which we were dealing. By the term "asymptotic instability" we mean the loss of stability for sufficiently high values of ω.

A simple way of overcoming this difficulty is to use the definition of stability in the sense of Liapounoff but, before formulating it, it is useful to make the following remarks.

In any physical system in a position of a stable equilibrium, the position of the representative point R never coincides *exactly* with the stable singular point S, but wanders slightly around S under the effect of some random disturbances inevitable in any such system. If one draws a small circle C around S as a center of radius greater than the greatest disturbing deviation (in a statistical sense), nothing is known about the motion of R inside C. On the circle C itself the situation is definite and one can formulate the criteria of stability in the following form.

1. If it is possible to show that through every point C the integral curves *enter* C and none of them leaves it, the equilibrium is stable.

2. If the integral curves leave C through every point of C and none of them enters it, the equilibrium is unstable.

In some special cases when, in addition to the singular point S, there exists a critical threshold traversing C, there is an additional definition.

3. If through some points of C the integral curves enter and through some other points leave the circle, the equilibrium is unstable.

It is precisely this third form of definition which concerns us here.

In fact, consider the characteristic equation (19) in which we set $(3r_0^2 - p) = N$; $s(\omega)\, N = M$. It becomes

$$S^2 + N\,S + M = 0. \tag{22}$$

A convenient way of analyzing the nature of the roots (and, hence, of singular points) is to draw in the plane of the variables M and N a

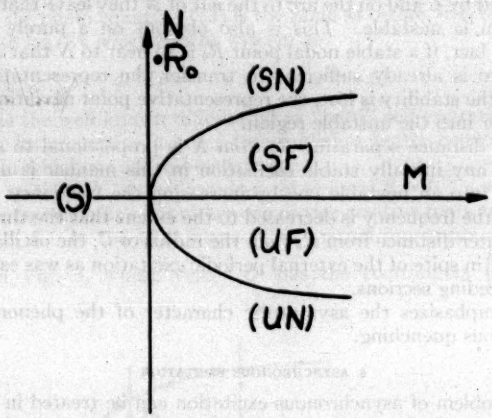

FIG. 1.

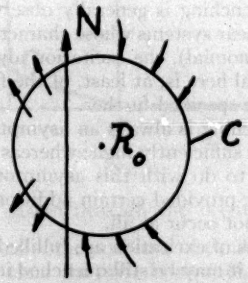

FIG. 2.

parabola V of equation $N^2 = 4M$ as shown in Fig. 1. The parabola and the axes M and N determine five regions (SN), (SF), (UF), (UN) and (S) corresponding to stable nodal, stable focal, unstable focal, un-

stable nodal and saddle points, respectively, the latter being located in the half-plane to the left of the N axis.

Since the quantity N is positive and finite and $s(\omega)$ approaches zero as ω increases, in the asymptotic case the singular point R_0 is a stable nodal point approaching the axis N (Fig. 1) indefinitely. For the sake of clarity the neighborhood of the point R_0 is shown in Fig. 2 on a larger scale.

Since on the arc of C to the right of N the integral curves enter the area limited by C and on the arc to the left of N they leave that area, the equilibrium is unstable. This is also obvious on a purely intuitive basis. In fact, if a stable nodal point R_0 is so near to N that a random disturbance is already sufficient to transfer the representative point across N, the stability is lost, the representative point deviating further and further into the unstable region.

As the distance separating R_0 from N is proportional to $s(\omega)$, it is clear that any initially stable oscillation in this manner is ultimately converted into an unstable one by increasing the frequency ω. Conversely, if the frequency is decreased to the extent that the threshold N is at a greater distance from R_0 than the radius of C, the oscillation will still persist in spite of the external periodic excitation as was established in the preceding sections.

This emphasizes the asymptotic character of the phenomenon of asynchronous quenching.

6. ASYNCHRONOUS EXCITATION

The problem of asynchronous excitation can be treated in a similar manner although the physical nature of the two phenomena is entirely different as was previously mentioned.

First, while the quenching is generally observed in "soft" systems (that is, in such non-linear systems whose characteristic can be approximated by a cubic polynomial), the excitation always occurs in "hard" systems (the polynomial here is, at least, of the fifth degree or higher). This, however, must be specified further.

Secondly, the quenching is always an asymptotic process occurring only if the frequency is sufficiently high; whereas the excitation, on the contrary, has nothing to do with this asymptotic condition and can occur at any frequency provided certain additional conditions are fulfilled. If not, it may not occur at all.

Finally, if conditions of excitation are fulfilled and the oscillation is thus brought into play, it may be still quenched in the same asymptotic manner in which is quenched the oscillation of a "soft" system.

We proceed now with a more detailed analysis of the excitation effect.

As was mentioned in the introduction, the differential equation of the "hard" excitation is governed by the differential equation of the form

$$\ddot{x} - (a + cx^2 - ex^4)\,\dot{x} + x = 0 \qquad (23)$$

where we assume provisionally that a, c and e are small positive numbers. The application of the stroboscopic method to (23) results in the differential equation

$$dr/d\tau = -\sigma'(r^5 - \alpha r^3 - \beta r) \tag{24}$$

$$d\phi/d\tau = gR/r \tag{25}$$

where $\alpha = 2c/e$; $\beta = 8a/e$; $h' = 8\gamma/\pi c(\omega^2 - 1)$; $E = e/\epsilon$; $g = \Gamma/2\pi \times (\omega^2 - 1)$; $\sigma' = +E/12$, the functions Q and R being the same as before.

Since the phase equation (25) is the same as before, it is sufficient to investigate Eq. 24 which can be written as

$$dr/d\tau = -\sigma' r(r^4 - \alpha r^2 - \beta). \tag{26}$$

This is the well known differential of "hard" self-excitation and we state here its properties in order to establish a better connection with the case in which we are interested here.

The stationary state requires $\rho^2 - \alpha\rho - \beta = 0$, $\rho = r^2$. This quadratic equation in ρ has either one or two positive roots according to the sign of β. In the (β,ρ) plane this equation represents a parabola, shown in Fig. 3, whose vertex O' has the coordinates $\left(\beta_1 = -\frac{\alpha^2}{4},\right.$

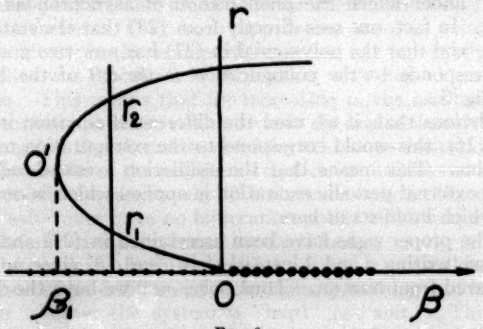

FIG. 3.

$\left.\rho_1 = \frac{\alpha}{2}\right)$ and which passes through the origin O. Since the abcissa axis represents, obviously, the positions of equilibrium (that is, singular points), one obtains thus the well known (4, Andronow) *bifurcation diagram*. We call to mind these conclusions: to the left of the *bifurcation point* O the topological configuration consists of stable singular points (shown in black points along the abcissa axis), surrounded by an unstable limit cycle r_1 which, in turn, is surrounded by a stable cycle r_2. This configuration exists only for negative values of β. For $|\beta| =$

$|\beta_1|$, r_1 and r_2 coalesce and disappear to the left of the point O'. At the point $\beta = 0$, the parameter β passes through the bifurcation value for which the configuration changes according to the scheme

unstable limit cycle
stable singular point } unstable singular point.

The unstable singular points are shown by circles to the right of O on the abcissa axis. In this region one has a familiar configuration of an unstable singular point surrounded by a stable limit cycle and the system becomes self-excited on the cycle r_2.

It is seen thus that, if one wishes to represent the phenomenon of asynchronous excitation in its initial phase (that is, if $\gamma = 0$, there is no oscillation), the coefficients β and a in (26) and (23) respectively must be negative so that these differential equations are now

$$\ddot{x} + (|a| - cx^3 + ex^4)\,\dot{x} + x = 0 \tag{27}$$

$$dr/d\tau = -\sigma'\Gamma(r^4 - \alpha r^2 + |\beta|). \tag{28}$$

The form of these differential equations is precisely that "special condition" under which the phenomenon of asynchronous excitation can occur. In fact, one sees directly from (28) that the state of rest is now stable and that the polynomial in (27) has now two positive roots which corresponds to the configuration *to the left* of the bifurcation point in Fig. 3.

It is obvious that, if we used the differential equation in the form (23) and (26), this would correspond to the configuration to the right of this point. This means that the oscillation *exists already* ($\gamma = 0$) before the external periodic excitation is applied which is certainly not the case which interests us here.

Once the proper signs have been ascertained in (27) and (28), one can continue writing a and β instead of $|a|$ and $|\beta|$ since no confusion is to be feared from now on. Thus, for $\gamma = 0$ we have the differential equation

$$dr/d\tau = -\sigma'(r^5 - \alpha r^3 + \beta r). \tag{29}$$

If an external periodic excitation is applied ($\gamma \neq 0$), the polynomial in (29) acquires a negative term, $-v(\omega)$, and one gets

$$dr/d\tau = -\sigma'(r^5 - \alpha r^3 + \beta r - v(\omega)) \tag{30}$$

and it is seen that the state of rest is now unstable which shows that the singular point is now to the right of the bifurcation point.

It remains to show now that the polynomial in (30) has positive roots. It is seen that at least one positive root exists, since the poly-

nomial is of an odd degree and its last term is negative. It is impossible, however, to know whether the equation of the fifth degree has other positive roots unless the coefficients are given numerically. This question is, however, of not much importance here since the self-excited oscillation establishes itself on the innermost limit cycle corresponding to the smallest positive root. Hence, as long as one knows that there exists at least one positive root, the phenomenon of asynchronous excitation is explained.

Summing up, this phenomenon finds its explanation in the shift of the singular point from the region in which the topological configuration is stable singularity, unstable limit cycle, stable limit cycle (that is to the left of the point O in Fig. 3), to another configuration of unstable singularity, stable limit cycle (to the right of the bifurcation point).

As to the asymptotic behavior of this case, it is exactly the same as that of the soft excitation studied in Section 5.

In fact, if one forms the characteristic equation corresponding to the variational equations of this case one obtains

$$S^2 + [\sigma' (5r_0^4 - 3\alpha r_0^2 + \beta)]S + \sigma' (5r_0^4 - 3\alpha r_0^2 + \beta)s(\omega) = 0. \quad (31)$$

By the condition of stability one ascertains that the bi-quadratic polynomial is positive and finite, whereas the function $s(\omega)$ is exactly the same as before. That is, it approaches zero as ω increases indefinitely. Hence in the asymptotic case one has an exactly the same situation that was discussed in connection with Figs. 1 and 2 of the "soft" case. This means that for increasing ω, the oscillation is ultimately quenched just in the same way as in the case of the soft excitation.

Summing up, the phenomenon of asynchronous excitation depends only on the transfer of the position of equilibrium across the bifurcation point and this is the reason why it is observed only in "hard" systems since the "soft" ones have no bifurcation point. The frequency of the external periodic excitation is not involved here.

Once the oscillation is thus caused to appear, it can still be quenched as in the case of an ordinary asynchronous quenching and here it does not matter whether the system is "hard" or "soft." This effect of frequency in the "hard" systems can only quench the existing oscillation but cannot cause its appearance from the state of rest.

The two phenomena, quenching and excitation, in "hard" systems thus occur simultaneously, but still their physical natures are entirely different.

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Wind Currents Explained.—The mechanism by which energy from the sun is converted into the vast wind currents which swirl throughout our atmosphere has now for the first time been explained.

Dr. Hsiao-lan Kuo, a native of China now working in meteorological research at the Massachusetts Institute of Technology, has succeeded in demonstrating, by means of purely mathematical and physical considerations, why the broad features of the circulation of the atmosphere assume the form they do.

Meteorologists have long known that the circulation of the atmosphere is maintained by the action of the sun, but they have never before been able to show just how this action takes place. "Dr. Kuo's outstanding achievement makes it possible to explain for the first time how the transformation from radiant energy to air motion is accomplished," says Dr. Victor P. Starr, Associate Professor of Meteorology, who reported Dr. Kuo's findings.

Professor Starr explains that radiant energy from the sun creates density differences in the atmosphere which represent a form of potential energy. This energy is in turn converted into the energy of atmospheric disturbances which range from the familiar storms associated with our daily weather changes to meandering "jet streams" encountered by pilots at great elevations.

Upper air observations, Dr. Starr points out, now reveal such strange features as these meandering rivers of air (known to meteorologists and pilots as jet streams) in middle latitudes with velocities up to 350 miles per hour, imbedded in broad high-level currents which advance from west to east around the globe. Nearer the equator, a vast ocean of air is observed to drift in the opposite direction (from east to west). Out of an infinite number of possible atmospheric disturbances only these familiar types are destined to utilize the potential energy from the sun for their growth. The rest never develop.

"Once the cyclones and jet streams have been created they tend to orient themselves in such a way as to maintain the broad westerly currents observed in middle latitudes and the easterlies in low latitudes," says Dr. Starr.

"They also accomplish the required energy transport from the tropics, where the heating from the sun is greatest, to the polar regions which would otherwise be at temperatures much below those we observe."

Dr. Kuo's work is part of a longer range program in the M.I.T. Meteorology Department sponsored by the Air Force Cambridge Research Center. This program was initiated in 1947 by Professor Starr with the purpose of coordinating and interpreting the tremendous mass of observational information about the atmosphere which has become available since the end of World War II.

Dr. Starr points out that this work cannot be expected to provide immediate practical aid in such matters as the prediction of hurricanes or the artificial control of the weather. "But its basic importance for practically every branch of meteorology can scarcely be overestimated," he declares.

THEORY FOR A SLIDER BEARING WITH A CONVEX PAD SURFACE; SIDE FLOW NEGLECTED

BY

STANLEY ABRAMOVITZ [1]

ABSTRACT

The effect of pad-surface curvature on load capacity, center of pressure, and fluid friction is determined using Reynolds' differential equation in hydrodynamic lubrication theory. A dimensionless parameter which is a function of the magnitude of curvature is introduced. Families of curves are obtained and presented in a manner similar to Norton (1).[2] The results indicate the practical use of pad curvature for centrally pivoted pads where the variation in fluid viscosity from pad inlet to outlet is negligible.

INTRODUCTION

It is the purpose of this study to demonstrate theoretically the effect of pad curvature on the operating conditions of a slider bearing.[3] Norton (1) presents, in a very clear manner, equations and resultant curves which describe slider bearing characteristics (load capacity, center of pressure, and fluid friction) as a function of the ratio of the inlet to the outlet film thickness. This study follows the same manner of presentation and results in families of curves with the magnitude of curvature as the parameter.

NOTATION

The following notation is used in this paper (see Fig. 1):

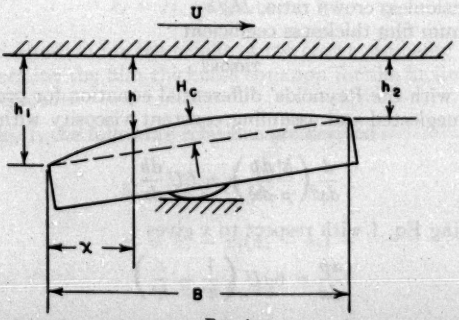

FIG. 1.

[1] The Franklin Institute Laboratories for Research and Development, The Franklin Institute, Philadelphia, Pa.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] Kingsbury or Michell type.

a—h_1/h_2, ratio of inlet to outlet film thickness
B—Breadth of pad in the direction of motion
c, c_1—Constants of integration
C_P—Dimensionless load factor for outlet film thickness
C_{P1}—Dimensionless load factor for minimum film thickness
C_e—Dimensionless support factor
C_f—Dimensionless friction factor for outlet film thickness
C_{f1}—Dimensionless friction factor for minimum film thickness
F—Resisting frictional force on slider
f—Film thickness function
H_e—Maximum height of pad crown (curvature) segment located at $B/2$
h—Film thickness
h_1—Inlet film thickness
h_2—Outlet film thickness
h_e—Height of pad crown segment
h_f—Film thickness for flat surfaced pad
h_m—Minimum film thickness
L—Length of pad perpendicular to motion
p—Fluid pressure
q—Pressure function
s—Shear stress
U—Surface velocity of slider
W—Bearing load
x—Coordinate
x_1—Dimensionless coordinate, x/B
$\bar{x}$—Distance to center of pressure (support pivot)
μ—Absolute viscosity
ϕ—Dimensionless crown ratio, H_e/h_2
ψ—Minimum film thickness coefficient

THEORY

Starting with the Reynolds' differential equation for pressure when side flow is neglected and assuming constant viscosity with respect to x, then

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx}. \tag{1}$$

Integrating Eq. 1 with respect to x gives

$$\frac{dp}{dx} = 6\mu U\left(\frac{1}{h^2} - \frac{c}{h^3}\right). \tag{2}$$

The film thickness, h, may then be obtained as a function of x (Fig. 1). This may be accomplished in the following manner:

(a) Determine the equation for the height of the crown segment. The curved surface was first assumed to be a portion of a circle. It

was found (see Appendix) that for small values of H_c, on the order of 10^{-3}, the equation for a parabola is a very good approximation (see Fig. 2).

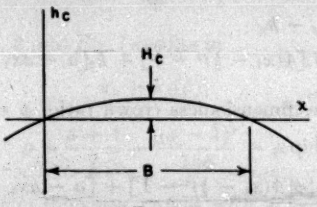

FIG. 2.

$$\left(x - \frac{B}{2}\right)^2 = k(h_e - H_e),$$

since

$$h_e = 0 \quad \text{when } x = 0$$

$$k = -\frac{B^2}{4H_e}$$

and

$$h_e = H_e - \frac{4H_e}{B^2}\left(x - \frac{B}{2}\right)^2. \tag{3}$$

Letting $x_1 = x/B$, Eq. 3 becomes

$$h_e = H_e - 4H_e(x_1 - \tfrac{1}{2})^2, \tag{4}$$

which is the equation for the height of the crown segment.

(*b*) Determine the film-thickness equation for the inclined chord of the crown segment.

From Fig. 1, the following relations are derived:

$$h_f = h_1 - \frac{x}{B}(h_1 - h_2)$$

$$h_f = h_1 - x_1(h_1 - h_2).$$

Let

$$a = h_1/h_2.$$

Therefore, by substitution, the film-thickness equation for the inclined chord of the crown segment is

$$h_f = h_2(a - ax_1 + x_1). \tag{5}$$

(*c*) Determine the final film-thickness equation.

If the pad inclination is assumed small, Eq. 4 when subtracted from Eq. 5 will result in the film thickness as a function of x_1, h_2, a, and H_e.

$$h = h_f - h_e$$

$$h = H_e[4(x_1 - \tfrac{1}{2})^2 - 1] + h_2[a - ax_1 + x_1]. \tag{6}$$

Introducing the dimensionless crown ratio, $\phi = \dfrac{H_e}{h_2}$, results in the following relationship:

$$h = h_2\{\phi[4(x_1 - \tfrac{1}{2})^2 - 1] + [a - ax_1 + x_1]\}, \tag{7}$$

which is the desired film-thickness equation.

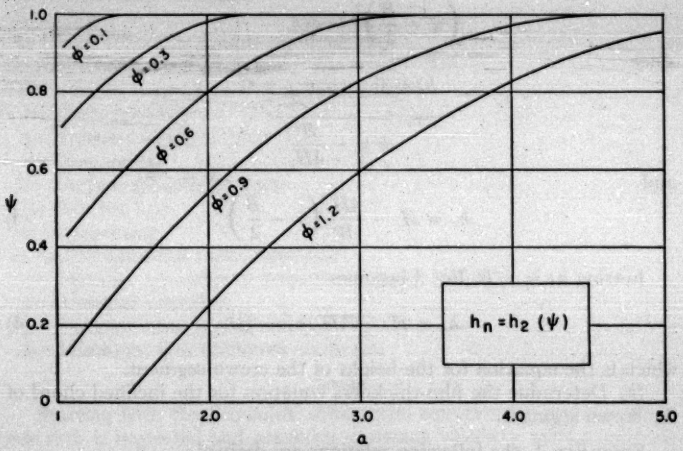

FIG. 3. Minimum film thickness.

OPERATING FACTORS

(*a*) *Minimum Film Thickness*

It is observed that for a convex surfaced pad, the minimum film thickness is not necessarily the outlet film thickness h_2, as is the case for a flat pad surface. The minimum film thickness which depends on the maximum height of the crown segment and the pad inclination may be obtained by differentiating Eq. 7 with respect to x_1 and equating to zero, as follows:

$$\frac{dh}{dx_1} = 8x_1\phi - 4\phi - a + 1 = 0.$$

Solving for x_1, one obtains

$$x_1 = \frac{a-1}{8\phi} + \frac{1}{2}. \tag{8}$$

Substituting Eq. 8 into Eq. 7 results in

$$h_n = h_2(\psi), \tag{9}$$

where

$$\psi = \frac{a+1}{2} - \frac{(a-1)^2}{16\phi} - \phi.$$

The minimum film-thickness curves with ϕ as the parameter are shown in Fig. 3.

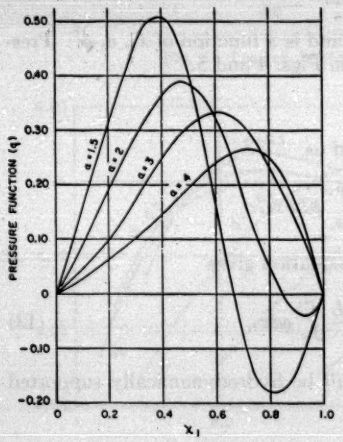

FIG. 4. Pressure distribution: $\phi = 0.6$.

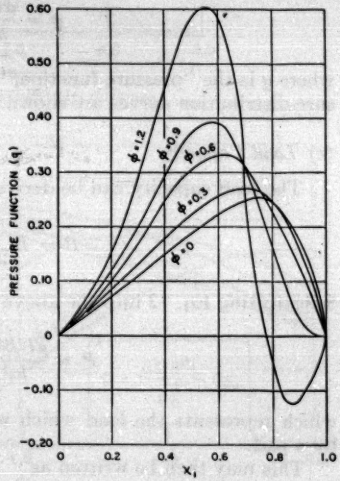

FIG. 5. Pressure distribution: $a = 3.0$.

(*b*) *Pressure Distribution*

From Eq. 2

$$\frac{dp}{dx_1} = 6\mu UB\left(\frac{1}{h^2} - \frac{c}{h^3}\right), \tag{10}$$

since

$$x_1 = \frac{x}{B}; \quad dx = Bdx_1.$$

Substituting Eq. 7 into Eq. 10 and integrating, one obtains

$$p = \frac{6\mu UB}{h_2^2}\left[\int \frac{dx_1}{f^2} - \frac{c}{h_2}\int \frac{dx_1}{f^2}\right], \tag{11}$$

where

$$f = \phi[4(x_1 - \tfrac{1}{2})^2 - 1] + [a - ax_1 + x_1]$$

or

$$f = 4\phi\left[x_1^2 + \frac{(1 - a - 4\phi)}{4\phi}x_1 + \frac{a}{4\phi}\right].$$

The constants c and c_1 (resulting from the integration of Eq. 11) may then be evaluated from the boundary conditions; $p = 0$ when $x_1 = 0$ or 1. The pressure distribution is then

$$p = \frac{\mu UB}{h_2^2}[q], \tag{12}$$

where q is the "pressure function," and is a function of x_1, a, ϕ. Pressure-distribution curves are shown in Figs. 4 and 5.

(*c*) *Load Capacity*

The load capacity can be derived as

$$P = B\int_0^1 pL dx_1.$$

Substituting Eq. 12 into the above equation gives

$$P = \frac{\mu UB^2L}{h_2^2}\int_0^1 q dx_1, \tag{13}$$

which represents the load which will be hydrodynamically supported by a pad.

This may then be written as

$$P = \frac{\mu UB^2L}{h_2^2}C_P \tag{14}$$

or, using the minimum film thickness,

$$P = \frac{\mu UB^2L}{h_m^2}C_{P1}, \tag{15}$$

where

$$C_{P1} = (\psi)^2 C_P.$$

The load coefficients, C_P and C_{P1} are dimensionless and are a function of a and ϕ. Load-coefficient curves are shown in Figs. 6 and 7.

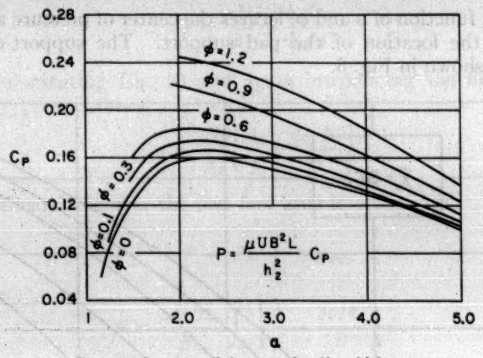

FIG. 6. Load coefficient: outlet film thickness.

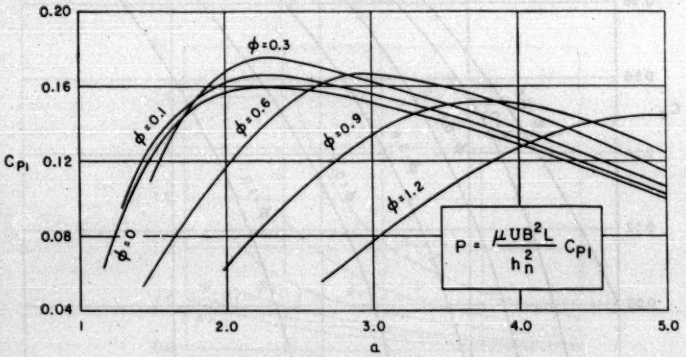

FIG. 7. Load coefficient: minimum film thickness.

(*d*) *Center of Pressure*

The center of pressure can be derived as

$$\bar{x} = \frac{B^2 L \int_0^1 x_1 p dx_1}{BL \int_0^1 p dx_1}. \tag{16}$$

Substituting Eq. 12 into Eq. 16 and integrating, gives

$$\bar{x} = BC_e, \tag{17}$$

where C_e, a function of a and ϕ, locates the center of pressure and therefore gives the location of the pad support. The support-coefficient curves are shown in Fig. 8.

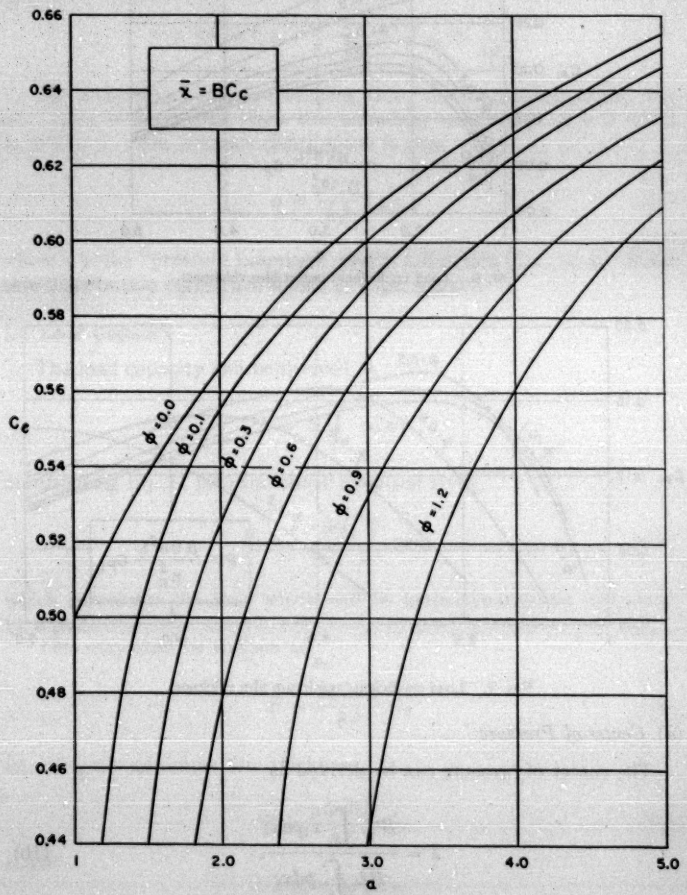

FIG. 8. Support coefficient.

(e) *Frictional Force*

The shear stress acting on the slider is given as

$$s = \frac{\mu U}{h} + \frac{h}{2}\frac{dp}{dx}.$$

By substituting Eq. 10 and again introducing the film-thickness function, f, one obtains

$$s = \frac{\mu U}{h_2}\left[\frac{4}{f} - \frac{3c/h_2}{f^2}\right].$$

The shear force over the pad-face area is

$$F = BL\int_0^1 s\,dx_1.$$

Hence

$$F = \frac{\mu UBL}{h_2}\int_0^1\left[\frac{4}{f} - \frac{3c/h_2}{f^2}\right]dx_1 \tag{18}$$

or

$$F = \frac{\mu UBL}{h_2}C_f. \tag{19}$$

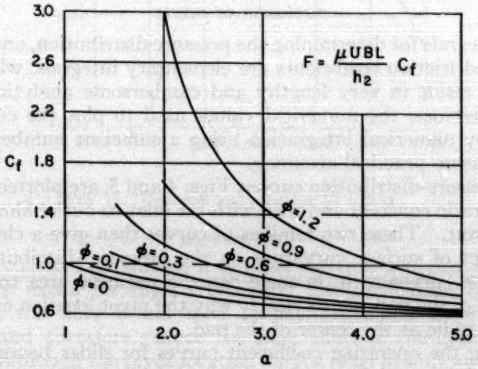

FIG. 9. Friction coefficient: outlet film thickness.

Using the minimum film thickness,

$$F = \frac{\mu UBL}{h_n}C_{f1} \tag{20}$$

where

$$C_{f1} = (\psi)C_f.$$

The friction coefficients C_f and C_{f1} are dimensionless, as for the load and support coefficients, and are a function of a and ϕ. Friction-coefficient curves are shown in Figs. 9 and 10.

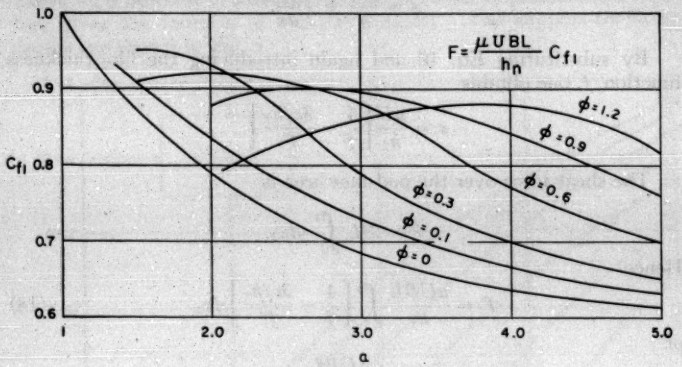

FIG. 10. Friction coefficient: minimum film thickness.

DISCUSSION OF CURVES

The integrals for determining the pressure distribution, and the load, support and friction coefficients are elementary integrals, which, when evaluated, result in very lengthy and cumbersome analytical expressions. Therefore, the numerical values used to plot the curves were obtained by numerical integration using a sufficient number of increments to insure practical accuracy.

The pressure-distribution curves, Figs. 4 and 5, are plotted first with the crown-ratio constant and then with the inlet-to-outlet film-thickness ratio constant. These two families of curves then give a clear picture of the effect of surface curvature on the pressure distribution. The shape of the curves with, in some cases, a negative area towards the outlet edge of the pad, shows clearly why the pivot location or center of pressure may lie at the center of the pad.

In using the operating coefficient curves for slider bearing design, it is possible to make use of optimum load capacity or power loss for minimum film thickness, using the curves in Figs. 7 and 10. This will give a crown ratio and inlet-to-outlet film-thickness ratio. With this information the pivot location may be obtained from Fig. 8. It is observed from Fig. 7 that a crown ratio in the region of 0.3 is optimum for maximum load capacity.

Among the other ways of using the curves, one practical example is where the crown is already determined, possibly due to practical limita-

tions, and the pivot location is set (centrally pivoted pad, where the bearing must be capable of running in both directions). A crown ratio is assumed, and from Fig. 8 the ratio a is obtained. The load coefficient may then be found from Fig. 6, and the film thickness h_2 can then be obtained from Eq. 14. The value of the crown ratio is then compared to the value assumed, and a second, or a third approximation may then be made until the assumed and calculated values agree.

It must be remembered that this analysis is based on no side flow. Therefore, side-flow correction factors based on the pad length-to-width ratio, must be applied. Until these factors are obtained for a convex surface, the existing side-flow coefficients for a flat surfaced pad may be used, with reservation (2).

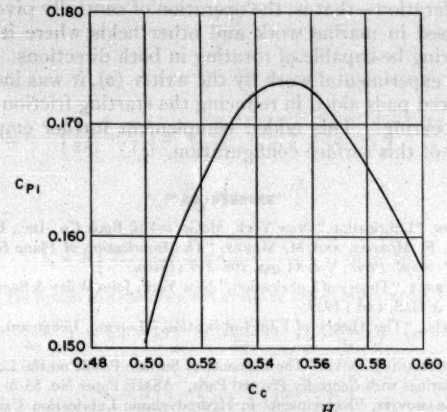

FIG. 11. Optimum load capacity: $\frac{H_2}{h_n} = 0.35$.

CONCLUSIONS

With a convex pad surface it is observed that a negative pressure theoretically exists for high crown and low inclination conditions. Practically, the fluid would tend to cavitate under these negative pressures, and therefore reduce the load effect of the negative area. Therefore, although the pivot location curves (Fig. 8) indicate values of the support coefficient less than 0.5, it is felt that the validity of the operating coefficients becomes questionable below this value.

A paper by Raimondi and Boyd (5) on the subject of pad-surface profile shows a curve of load coefficient *versus* the ratio of crown to minimum film thickness for a centrally pivoted pad. Using the curves in this paper, an extension was made of this approach and a family of curves with the pivot location as the parameter was obtained. It was interesting to note that for all support factors (pivot points) from 0.50

to 0.60 the maximum load capacity was at a crown to minimum film-thickness ratio of approximately 0.35. A curve of load coefficient *versus* support factor was then plotted for this optimum crown ratio. This curve is shown in Fig. 11, and indicates an optimum support factor of 0.55. The use of an optimum support factor and crown ratio would result in approximately a 10 per cent increase in load capacity as compared to a flat pad located at its optimum pivot point. It may be noted that this paper was developed independently of the work of Raimondi and Boyd, and basically substantiates their conclusions.

Aside from the effect of variable viscosity, the theory for a flat surfaced pad indicates zero load capacity when the pivot is located at the pad center. Therefore, in general this study does point to one major consideration—that is, the operation of centrally pivoted pads so commonly used in marine work and other fields where it is required that the bearing be capable of rotating in both directions.

Based on experimental work by the writer (6), it was indicated that convex surfaced pads aided in reducing the starting friction of a tilting-pad thrust bearing. This added complement further emphasizes the practical use of this surface configuration.

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(5) A. A. RAIMONDI AND J. BOYD, "The Influence of Surface Profile on the Load Capacity of Thrust Bearings with Centrally Pivoted Pads," ASME Paper No. 53-A-166.
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APPENDIX

Proof that the equation for a parabola as compared to that of a circle is a very good approximation for small values of H_c is as follows:

$$\left(x - \frac{B}{2}\right)^2 + (h_c + (R - H_c))^2 = R^2.$$

Therefore,

$$h_c^2 + 2(R - H_c)h_c + \left[\left(x - \frac{B}{2}\right)^2 - 2H_cR + H_c^2\right] = 0.$$

When

$$x = 0; \quad h_c = 0.$$

Therefore

$$R = \frac{B^2 + 4H_c^2}{8H_c}.$$

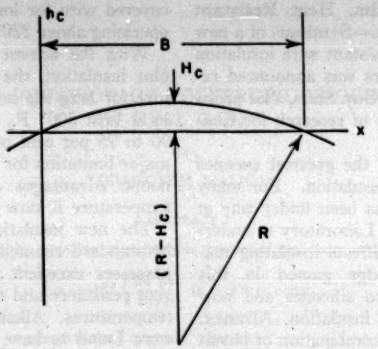

FIG. 12.

Then

$$h_c = -\left[\frac{B^2}{8H_c} - \frac{H_c}{2}\right] \pm \sqrt{\left(\frac{B^2}{8H_c} - \frac{H_c}{2}\right)^2 - \left(x - \frac{B}{2}\right)^2 + \frac{B^2}{4}}.$$

Let

$$x_1 = \frac{x}{B} \quad \text{and} \quad \alpha = \frac{H_c}{B}.$$

Eliminating H_c,

$$h_c = B\left[\frac{4\alpha^2 - 1}{8\alpha}\right] \pm B\sqrt{\left(\frac{1 + 4\alpha^2}{8\alpha}\right)^2 - (x_1 - \tfrac{1}{2})^2}.$$

Expanding the square root quantity and dropping higher order terms,

$$\left[\left(\frac{1 + 4\alpha^2}{8\alpha}\right)^2 - (x_1 - \tfrac{1}{2})^2\right]^{\frac{1}{2}} = \frac{1 + 4\alpha^2}{8\alpha} - \frac{1}{2}\left(\frac{1 + 4\alpha^2}{8\alpha}\right)^{-1}(x_1 - \tfrac{1}{2})^2.$$

Therefore

$$h_c = B\left[\alpha - \left(\frac{4\alpha}{1 + 4\alpha^2}\right)(x_1 - \tfrac{1}{2})^2\right].$$

Since α is on the order of 10^{-3},

$$1 + 4\alpha^2 \cong 1$$

and, substituting for α,

$$h_c = H_c - 4H_c(x_1 - \tfrac{1}{2})^2.$$

New Thin-Film, Heat Resistant Wire Insulation.—Synthesis of a new thin-film, heat-resistant wire insulation for electric motors was announced recently by Dr. C. Guy Suits, vice president and director of research for General Electric.

Heat is one of the greatest enemies of all electrical insulation. For many years a search has been under way at the GE Research Laboratory to understand how heat affects insulating materials. Knowledge gained in this search led first to silicones and now to the new wire insulation, Alkanex.

Alkanex has a combination of physical and chemical properties long sought by research chemists—properties that will allow engineers to increase the horsepower of motors without increasing their size. Even a small electric motor contains half a mile or more of finely-drawn insulated wire.

Insulation may be thought of as a fence, keeping electricity in its proper path. When such "fences" break down owing to a rough treatment, exposure to corrosive fumes or overloading, motor failures result. For many decades after the electric motor was invented, wire insulation consisted primarily of wrappings of cotton, silk, or paper with varnish. Motors were at least half again as large and heavy as they needed to be, if thinner "fencing material" had been available.

During the 1930's the problem was solved for ordinary temperature operations by the introduction of Formex insulated wire. Motor size and weight reductions were realized almost immediately. It was still necessary, however, to use bulky glass or asbestos-covered wire for long-lived equipment operating above 220° F.

With the advent of Alkanex thin-film insulation, the limiting temperature for long life now has been raised to at least 300° F., which will include 90 to 95 per cent of all motors. The major limitation for exploiting the economic advantages of high operating temperature is now removed.

The new insulation can be applied by standard enameling equipment. It possesses excellent abrasion and solvent resistance and resists flow at high temperatures. Alkanex-insulated wires were found to have outstanding heat-resisting ability. Baked at 300° F. for more than six months, for example, there were no signs of deterioration or loss of insulating strength. By contrast, under the same conditions, conventional wire enamels had failed completely. Tests at even higher temperatures indicate we can expect electrical equipment to last many years at 300° F., or at least 75° higher than was previously possible.

Motors once rated at 20 h.p. may be upgraded to 25 h.p. or more, which means they will be able to carry a much bigger load.

Toughness and ability to resist abrasion are essential qualities needed by any magnet wire used in motors. The pulling, bending, and flexing of the wire during the construction of a motor puts a severe strain on the enamel as well as on the conductor. Insulation must not be broken or weakened. In experiments with Alkanex, the coated wire has been pounded flat without breaking the organic film.

THE EQUATION OF POLYTROPIC PROCESS OF REAL GASES

BY

DRAGOMIR MALIC [1]

ABSTRACT

In this paper the equation of polytropic process of real gases is derived as:

$$\left[p_s + 427\left(\frac{\partial u}{\partial v}\right)_T\right]\cdot(v-b)^n = \text{const.},$$

where $427\left(\frac{\partial u}{\partial v}\right)_T$ represents the "internal" pressure.

This equation applies to cases when the exponent n can be taken as a constant.

INTRODUCTION

In the well known term of the first law of thermodynamics relating to the *ideal gas*

$$dq = c_v dT + 1/427\cdot p\,dv, \tag{1}$$

the pressure p, according to the kinetic theory of gases, is proportional only to the mean translational kinetic energy of molecules.

In the case of *real gases* there is, apart from the kinetic energy of molecules, also the potential energy arising from the intermolecular forces (van der Waals).

Therefore, the first law of thermodynamics relating to real gases can be stated:

$$dq = c_v dT + 1/427\left[427\left(\frac{\partial u}{\partial v}\right)_T + p_s\right]\cdot dv, \tag{2}$$

where p_s is the measured pressure of the gas.

The expression $427\left(\frac{\partial u}{\partial v}\right)_T\cdot dv$ in Eq. 2 represents actually the necessary work to oppose the action of the intermolecular forces. If the corresponding equation of state is known, the value of that work can be obtained from the following equation:

$$427\left(\frac{\partial u}{\partial v}\right)_T = T\cdot\left(\frac{\partial p_s}{\partial T}\right)_v - p_s. \tag{3}$$

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For instance, in case of the ideal gas (since $pv = RT$ and $p_s = p$) there is:

$$\left(\frac{\partial p_s}{\partial T}\right)_v = \left(\frac{\partial p}{\partial T}\right)_v = \frac{p}{T};$$

consequently from Eq. 3 there is:

$$427\left(\frac{\partial u}{\partial v}\right)_T = 0,$$

that is, the external work necessary to oppose the action of intermolecular forces in the ideal gas is equal to zero, the forces in question being negligible, according to the definition of the ideal gas.

In the case of the van der Waals' gas, however,

$$p_s + \frac{a}{v^2} = \frac{RT}{v - b}, \tag{4}$$

so that

$$\left(\frac{\partial p_s}{\partial T}\right)_v = \frac{R}{v - b}, \tag{5}$$

and

$$427\left(\frac{\partial u}{\partial v}\right)_T = \frac{a}{v^2}, \tag{6}$$

that is, the work necessary to oppose the action of intermolecular forces in the van der Waals' gas is equal to $\frac{a}{v^2} \cdot dv$.

Equation 2 for the van der Waals' gas becomes then:

$$dq = c_v dT + \frac{1}{427} \cdot \left(\frac{a}{v^2} + p_s\right) \cdot dv. \tag{7}$$

DEFINITION OF THE "INTERNAL" PRESSURE AND THE EQUATION OF THE POLYTROPIC PROCESS OF REAL GASES

If the equations of the polytropic process of the ideal gas,

$$pv^n = \text{const.}, \tag{8}$$

or

$$Tv^{n-1} = \text{const.}, \tag{8a}$$

were corrected in the same way that van der Waals has corrected Clapeyron's equation of state, the equations of the polytropic process of the

van der Waals' gas would be:

$$\left(p_e + \frac{a}{v^2}\right) \cdot (v - b)^n = \text{const.}, \tag{9}$$

or

$$T \cdot (v - b)^{n-1} = \text{const.} \tag{9a}$$

It follows from Eq. 9a that

$$\frac{dT}{T} + (n - 1)\frac{dv}{v - b} = 0;$$

consequently

$$dv = -\frac{1}{n - 1} \cdot (v - b) \cdot \frac{dT}{T}. \tag{10}$$

Comparing Eqs. 4, 7 and 10 we get:

$$dq = c_v \cdot \frac{n - \gamma}{n - 1} \cdot dT = c \cdot dT, \tag{11}$$

that is, the well known term for the quantity of heat of polytropic change of state of the ideal gas.

As shown, Eq. 11 can be applied both to the ideal gas and to the van der Waals' gas, where $c = c_v \cdot \frac{n - \gamma}{n - 1}$ is any specific heat. If the exponent of the polytropic curve n remains unchanged, as supposed in the above case, the value of c remains also constant, whereas if n varies, c varies too.

However, when comparing Eqs. 1, 2 and 7 we get:

$$p = p_e + 427\left(\frac{\partial u}{\partial v}\right)_T = p_e + \frac{a}{v^2} = p_e + p_i. \tag{12}$$

The term $a/v^2 = p_i$ represents the "internal" pressure of the van der Waals' gas. Generally, in the case of each individual real gas the "internal" pressure can be defined as:

$$p_i = 427\left(\frac{\partial u}{\partial v}\right)_T, \tag{13}$$

and it can be solved, as shown above in the case of both the ideal gas and the van der Waals' gas, using expression (3), and knowing the equation of state of the gas in question.

The equation of the polytropic process of real gases can consequently be given in a general form:

$$\left[p + 427\left(\frac{\partial u}{\partial v}\right)_T\right]\cdot(v-b)^n = \text{const.} \qquad (14)$$

This equation applies to cases when the exponent n can be taken as a constant.

Experience shows that actual thermodynamic processes, such as in the case of heat engines, can be represented approximately by a polytropic curve lying between the isothermal and the adiabatic curves.

NUCLEAR ENERGY NEWS

Because of the increasing participation by industry in the nuclear energy program, the JOURNAL begins with this issue a monthly digest of significant new developments in this field that are not classified. It will be based on the official releases of the Atomic Energy Commission and its contractors.—Ed.

CONTRACT FOR ARMY PACKAGE POWER REACTOR

A lump-sum contract to design, build, and test-operate a prototype "package" nuclear power plant for military use was awarded by the AEC to the American Locomotive Company in December 1954. The contract is the first of this kind to be let on a fixed-price basis and as such is an important step in the development of power reactor technology.

Of 18 proposals submitted by industrial firms, the Company's was the most favorable to the Government from the standpoint of price, excellence of design, and responsiveness to other terms of the AEC's invitation. Twelve of the proposals were under $4,000,000 and the other six ranged upward to a high of $6,900,000. The contract price is $2,096,753, which was the lowest bid. This cost will be shared by the AEC and the Department of the Army on an approximately equal basis.

The plant, which has been designated the Army Package Power Reactor (APPR), will be built to provide construction, operation and maintenance information and to demonstrate both the capabilities and limitations of such plants. The design of the APPR is based on studies by the Oak Ridge National Laboratory which established the feasibility of developing such a plant.

The project is designed to develop a power reactor plant, with components transportable by air, which can be used at remote bases. Use of such a plant would reduce the amount of bulky conventional fuels that must be transported to support a military operation. The prototype will have a capacity of about 2000 kilowatts of electricity. Future plants based on this experiment could be built to provide space heating in addition to electricity. The prototype will be built at Fort Belvoir, Virginia, site of the Army Corps of Engineers Training Center, where it will provide a training facility which can be integrated into the engineer training program of the Army.

In commenting on the AEC award, Perry T. Egbert, ALCO President, had this to say:

ALCO has been building atomic energy products since 1949 and has supplied major components for the *USS Nautilus* and for basic atomic energy producing plants. The assignment of designing and building the first atomic energy package power plant, therefore, is a responsibility we welcome. This prototype of a complete power plant, which can be transported and erected anywhere in the world, should contribute materially to the objective of utilizing atomic energy commercially.

POWER DEMONSTRATION REACTOR PROGRAM

The Atomic Energy Commission in January inaugurated a Power Demonstration Reactor Program designed to open the way for American industry to develop, fabricate, construct and operate experimental nuclear power reactors. The aim of the program is to bring private resources into the development of engineering information on the performance of nuclear power reactors and to advance the time when nuclear power will become economically competitive.

Under this program proposals for Commission cooperation may be submitted up to April 1, 1955 by applicants who are willing to assume the risk of construction, ownership, and operation of reactors designed to demonstrate the practical value of such facilities for industrial or commercial purposes. The Commission may accept one or several of the proposals—or it may find that none is acceptable.

In evaluating the proposals the Commission will employ the following criteria:

a. Probable contribution of the proposed project toward achieving economically competitive power;
b. Cost to AEC in funds and materials;
c. Risk to be assumed by the maker of the proposal;
d. Competence and responsibility of the maker of the proposal;
e. Assurances given by the maker of the proposal against abandonment of the project.

Those proposals qualifying as acceptable under these criteria will be considered on a competitive basis within the limit of funds and materials available. The cooperation and assistance that the AEC will grant includes loan without charge of special nuclear materials and the performance of research and development work in AEC laboratories without charge.

Also the AEC will enter into research and development contracts for the technical and economic information resulting from development, construction, and operation of power demonstration reactors by contractors. The amounts to be paid by AEC under the contracts will be fixed in advance and may be used for development, fabrication, and experimental operation. Information derived from operations under the contracts will be made available by AEC to the maximum extent practicable to the entire technical public working on reactor development, with resultant benefit to the progress of the entire nuclear power program.

HAFSTAD RECEIVES AEC AWARD

Dr. Lawrence R. Hafstad, at about the time he resigned as Director of the Atomic Energy Commission's Division of Reactor Development, was granted the AEC's Distinguished Service Award. The award, es-

tablished in January 1954, is part of the AEC's incentive award program. It is granted in recognition of exceptionally meritorious and effective performance of duty by staff members. A citation, signed by the Commissioners, and a lapel button bearing the seal of the AEC, were presented by Chairman Strauss to Dr. Hafstad at a meeting of the Commission and the principal staff Tuesday, December 21, 1954.

In the citation Dr. Hafstad's achievements during his six years of service with AEC were listed as follows:

> Dr. Hafstad was responsible for the development of the Commission's reactor program and the completion of the first generation of post-war reactors. Under his guidance, the country's first electric power was produced from nuclear fuel at the National Reactor Testing Station. He pioneered and administered with the Department of Defense a program for the development of military nuclear power plants for the Navy, Air Force and the Army, including the first nuclear powered submarine, the *Nautilus*. He was instrumental in the formulation of a program of participation by industry, both technically and financially, in the development of nuclear power plants. Under his leadership, the Nation's first civilian nuclear power plant was recently placed under construction near Pittsburgh, Pennsylvania.

RADIOISOTOPE COURSE FOR FOREIGN SCIENTISTS

A special four-week course in radioisotope (tracer atom) techniques for scientists and technicians from the 53 countries qualified to receive shipments of U. S. manufactured isotopes will open on May 2, 1955. It will be identical with the training given six times a year by the Commission. Like the regular sessions it will be limited to 32 enrollees.

The special session is in line with other steps being taken by the Commission in support of President Eisenhower's atoms-for-peace program. Provisions will also be made for admitting a limited number of qualified foreign research workers in subsequent sessions of the training course given at the Oak Ridge Institute of Nuclear Studies. This is an educational corporation of 32 colleges and universities in the South under contract to the AEC.

The purpose of the Oak Ridge course is to permit qualified research workers and technicians to gain sufficient facility in the use of radioisotopes, or tracer atoms, to apply this important technique in their own work. During the four weeks of intensive training, basic fundamentals in radioisotope use are taught. The participants learn how to use and calibrate radiation detection instruments, how to purify and separate radioactive materials from inert or other radioactive materials, and how to apply them to a variety of chemical and biological research problems. Background lectures in nuclear physics and chemistry and special-topic seminars are combined with laboratory work.

The approximately 2000 men and women who have received this training since it began in 1948 have found it useful in a wide range of scientific, medical, agricultural, and industrial areas, although completion of the course does not qualify physicians to use radioisotopes in

general practice. Prospective enrollees must have a bachelor's degree; adequate training and experience in the field in which they propose to use isotopes; and an understanding of the English language.

UNCLASSIFIED PHYSICAL RESEARCH CONTRACTS

Award of 49 unclassified physical research contracts with universities and private research institutions was announced last December and January by the U. S. Atomic Energy Commission. Most of them are renewals of contracts which have been in force. They are generally for a term of one year. These contracts are part of the AEC's continuing policy of utilizing private research laboratories in conducting research related to atomic energy.

Proposals for these contracts are usually reviewed by scientists who are well acquainted with the field of the proposed research and its relevance to the atomic energy program. Their reviews are taken into consideration by the AEC Division of Research before a decision is made whether or not to accept the proposal. The contracts are negotiated and administered by AEC Operations Offices.

Generally, the institutions participate with the AEC in defraying the costs of the research. The institution contributes the funds and services which it normally devotes to work in that field, and the AEC provides additional assistance to permit the work to proceed more rapidly or with an expanded scope. One-third of the grants were for amounts between $4000 and $10,000, one-third between $10,000 and $20,000, and one-third for amounts up to $450,000.

ALPHA-RADIATION FLOOR MONITOR

The University of California's Los Alamos Scientific Laboratory has announced that a floor monitor has been designed to detect radioactive contamination. It is a portable, battery-operated instrument which is pushed over the floor like a vacuum cleaner.

A basic box unit mounted on three wheels serves as a rolling bed for an alpha-sensitive probe and associated power supplies and amplifiers. A control box mounted on a three-foot long handle contains a reset switch, a range switch, a loudspeaker to provide audible signals of alpha counts, and a counting rate meter. The entire instrument is designed for easy disassembly and repair.

The new monitor, designed by Mark H. Tattan of the Laboratory's Chemistry and Metallurgy Division, was developed specifically for monitoring the floor areas of that Division's new research building. It has proved to be extremely useful and convenient for detecting radioactive materials of the alpha emitting variety. It is a great improvement over the past practice of having a monitoring technician crawl on his hands and knees while holding a small-area probe at a fixed distance from the floor.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS

ADHESIVE BONDED AIRCRAFT STRUCTURES

The development of high strength metal-to-metal adhesives in recent years has made possible an entirely different approach to the fabrication of metal aircraft structures. Certain economic, structural and weight-saving advantages offered by the substitution of adhesives for rivets have already been realized by several aircraft companies in the fabrication of structural components, and the widespread adoption of this method in the future has become a definite possibility. To secure a better understanding of the behavior of adhesive bonded aircraft structures, Wilhelmina D. Kroll and Leonard Mordfin of the National Bureau of Standards have conducted investigations under the sponsorship of the National Advisory Committee for Aeronautics and the Navy Bureau of Aeronautics.

The use of adhesives for bonding aircraft structural components is not new. During World War I casein was used in the gluing of wooden aircraft structures. The synthetic glues were used extensively for many all-wood and some wood-metal joints for aircraft in World War II. Finally, the development of high strength resin and resin-rubber adhesives made possible the bonding of all-metal aircraft structural components. Two such components are the sandwich panel and the sheet-stringer panel.

A sandwich panel consists of three laminations bonded together. The outer two laminations, or facing, are made of strong, stiff, sheet materials which usually determine the elastic and strength properties of the panel. The central layer, or core, is made of a light material and serves only to separate the facings and to restrain them from becoming elastically unstable. The value of sandwich construction lies in its relatively light weight and in the high bending strength and stiffness it possesses as a result of its high ratio of cross-sectional moment of inertia to weight. Another desirable advantage (from the aerodynamic standpoint) is the absence of rivet heads.

One of the applications of sandwich construction is in the fabrication of skins and floors for pressurized aircraft fuselages where the sandwich construction is subjected both to lateral pressures and to axial loads simultaneously. The Bureau's engineering mechanics laboratories conducted an investigation for the NACA to determine the strength of sandwich panels of various thicknesses under these combined axial and lateral loads.

The panels tested were about 30 in. long and 17 in. wide, with facings of either 0.025- or 0.032-in. 75S-T6 aluminum alloy sheet, and cores ½ or ¾ in. thick. The cores are honeycombs of 0.005-in. 2S-H aluminum foil, having a density of only 6.54 lb. per cubic foot.

The lateral pressures were applied by a rubberized fabric air bag and ranged from zero to 9.54 psi. The panels were simply supported in knife edge seats in a hydraulic testing machine which applied the axial load. All tests were carried to failure, and strain and deflection measurements were made periodically with SR-4 wire strain gages and dial gages, respectively.

The results of these tests were compared with values predicted from a theoretical analysis based on column theory. The analysis includes the effects of the strain energy stored in the faces of the panels by extension and by bending, and in the core by shear, but disregards the strain energy due to shear in the faces and to extension in the core parallel to the faces. Good agreement was obtained between theory and experiment for the flexural rigidity and for the failing loads under the various load combinations investigated. The theory was found to be conservative, however, in its prediction of axial strains and lateral deflections. The discrepancy was attributed to anticlastic bending, which was observed in the tests but does not enter into the column theory. Anticlastic bending produces a transverse curvature in a panel which increases its stiffness and thus reduces the strains and deflections for a given loading condition. The amount of anticlastic bending obtained in a panel increases with the lateral load and decreases with panel stiffness. Hence the agreement between experiment and theory was considerably closer for the stiffer panels tested and for the panels tested under the lesser lateral pressures. For a panel tested with zero lateral pressure, the agreement was excellent.

When a sandwich panel is attached to some other structural member, it may be necessary to transfer the loads from the member to the panel by means of a pin that fits into a reinforced hole in the panel. As part of an earlier research project conducted by NBS for the Navy Bureau of Aeronautics, a test was performed on a sandwich panel to determine the stress distribution near a reinforced circular hole in the panel that was loaded by a pin. The results indicate that if the reinforcement is bonded to the sheet so that reinforcement and sheet act as an integral unit, then plane stress theory will predict the average elastic stresses in both sheet and reinforcement and also the changes in the hole diameter.

A more conventional type of aircraft structure to which adhesives have been applied is the sheet-stringer panel. Sheet-stringer construction, which comprises most of the outer skin of a conventional airplane, consists of a sheet with stiffeners attached longitudinally to one side.

The sheet provides the necessary aerodynamic contour while the stiffeners, or stringers, add the necessary strength, stiffness and stability. The stringers often are angles or channels, but very frequently have cross-sections resembling Z's, I's or hats. In the past, the stringers were almost always riveted to the sheets. Two new innovations are the bonded sheet-stringer panel and the integrally machined sheet-stringer panel.

The Bureau tested twenty-one riveted and bonded sheet-stringer panels for the NACA to determine the relative merits of these two types of construction. The panels were fabricated from 75S-T6 aluminum alloy with five stringers each, spaced at 3½-in. intervals. Nine panels were riveted with a 1⅛-in. pitch, nine were bonded with an epoxy resin adhesive and three were bonded with a phenolic resin-rubber adhesive. Some of the panels were tested in flat end compression, others in compression with a concentrated load applied to one end of the center stringer, and the remainder in bending with a concentrated load applied midway between simple supports.

The results obtained in the tests show that the mechanical properties of both types of panels are comparable. The choice in any particular case would depend upon the specific designs being compared.

The primary advantage of bonded sheet-stringer panels over conventional panels which have one line of rivets per stringer is the increased sheet stability that is obtained by bonding. The broad area of attachment between sheet and stringers reduces the unsupported width of sheet, thus increasing the load necessary to produce buckling of the sheet between stringers. When buckling occurs, the broad area of attachment between sheet and stringers provides a greater effective sheet width. In addition, inter-rivet buckling and stress concentrations near the rivet heads which tend to produce buckling between stringers are eliminated.

The primary disadvantage of bonded sheet-stringer panels seems to be in the mode of failure in flat end compression. When maximum load was reached in the tests, the bonded panels experienced almost complete destruction. The failure of the riveted panels, on the other hand, was confined to local buckling of sheet and stringers between rivets.

One of the most important factors restricting the universal acceptance of adhesive bonding for aircraft structures is the difficulty of properly inspecting bonded joints. Small changes in pressure, temperature, humidity, etc., in the fabricating process can produce imperfections which greatly decrease the strength of the joint, and can often be detected only by destructive tests. These imperfections can be dangerous in modern high-speed aircraft which are designed with extremely high operating stresses. An important consideration, therefore, in the widespread application of bonding techniques is the development of

either completely reliable fabrication procedures or of satisfactory nondestructive inspection procedures.

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THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

February 16, 1955

The Stated Monthly Meeting of The Franklin Institute was held on February 16, 1955, in the Lecture Hall. In the absence of the President, Henry B. Allen, Executive Vice-President, called the meeting to order at 8:20 P. M. and acted as Chairman. Approximately 340 members and guests were in attendance.

The Chairman stated that the Minutes of the December Stated Meeting were printed in the January JOURNAL and asked if there were any corrections, additions, or changes. None being offered, they were approved as submitted. He advised that Minutes of the Annual Meeting held in January will be printed in the February JOURNAL and will be submitted for approval at the next Stated Meeting.

H. V. Bail, Assistant Secretary, reported that elections to membership, recorded since December 31, 1954, resulted in a net gain in the following classes of membership:

Student	5
Individuals with Library privileges under Company membership	7

and a total membership of 6831 as of January 31, 1955.

The Chairman introduced the speaker for the evening, Dr. W. F. G. Swann, Director of the Bartol Research Foundation of The Franklin Institute and Senior Advisor to the Institute's Laboratories for Research and Development. Dr. Swann's lecture, "The Known and the Unknown," was enthusiastically received by the audience and will be printed in the May issue of the JOURNAL.

Following a question period, the meeting adjourned at 9:30 P. M., with a rousing vote of thanks to Dr. Swann.

H. V. BAIL
Assistant Secretary

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

COULSON, THOMAS: Some Prominent Members of The Franklin Institute. 2. Joseph Saxton
FAUPEL, J. H.: Residual Stresses in Heavy-Wall Cylinders
KRON, GABRIEL: Detailed Example for Interconnecting Piece-Wise Solutions
LAMBERG, ARTHUR B.: Beams of Metal or Wood Subjected to Millions of Reversals of Compression and Tension or to Longtime Steady Compression Longitudinally
MOON, PARRY AND DOMINA EBERLE SPENCER: A Postulational Approach to Electromagnetism
FALES, ELISHA N.: A New Laboratory Technique for Investigation of the Origin of Fluid Turbulence

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, February 9, 1955.)

HALL OF THE COMMITTEE,
PHILADELPHIA, FEBRUARY 9, 1955.

PROFESSOR W. S. PARDOE in the Chair.

The following report was presented for final action:

No. 3265: Photon Type-Composing Machine.

This report recommended the award of a John Price Wetherill Medal, each, to René A. Higonnet and Louis M. Moyroud, both of Cambridge, Massachusetts, "For their conception and development of the Photon Type-Composing Machine."

JOHN FRAZER,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED BY THE BOARD OF MANAGERS

February 16, 1955

ACTIVE FAMILY

W. P. Eckfeldt — Robert W. Gift

ACTIVE

William A. Bender, III	Hans G. Figdor	D. R. MacDonnell
Frederic M. Bauer	Robert Glendenning Ives	Bradley V. M. O'Brien
John C. Bovenkerk	Norman Kramer	Abraham A. Perez
Mrs. R. W. Carey	Robert K. Lawhorn	Albert Quell
George E. Deming, Jr.	Jack Lutz	Clarence N. Smith
John O. Estlow		Jeremy O. Sweeten

ACTIVE NON-RESIDENT

J. R. Adams, Jr.	Cole A. Allen	O. C. Martenet

NECROLOGY

R. A. Gravdahl, '43	F. H. Martin '53	Charles H. Priestly '41
Jack Karnell '54	Ellice McDonald '28	A. Merrill Redding '44
Jack H. Landau '43		Henry Saur '43

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A. M. until 5 P. M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; 2 P. M. until 10 P. M. on Wednesdays.

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NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

DEDICATION OF THE NICKELODEON

In a recent Museum Note our readers were informed of an addition to the Museum in the form of a Nickelodeon. On Tuesday, January 18th, this room was formally presented to the Institute by the donor, Mr. William Goldman, as a tribute to the motion picture industry. At the preliminary ceremonies held at a luncheon in Franklin Hall, attended by numerous representatives of all branches of the industry, Mr. Goldman said:

"Assembled here in this historic institution are the men and women who represent every segment of the motion picture industry, the presidents of our large producing companies, together with the generals of their sales staffs, our top-flight producers, directors, our stars of the motion picture screen—our actors and actresses. It is a great honor to be able to stand here and bear testimony in the presentation of the replica of the turn of the century, which will be given to this institution for posterity. I travel many years in retrospect when I think of the great progress which has been made in the motion picture industry, from the silent flickers to the great esteem to which motion pictures have arrived, and this young industry, this growing youngster which will give a great account of itself as time progresses.

"I get a little irked when I learn that people think of us as just making pictures for entertainment. I think of what our industry has done in the field of education, in the school systems, the colleges, and the universities. I am also reminded of the great value pictures have been to the Armed Forces in the dissemination of information in the prosecution of the war effort, and I know personally of what its contribution has been in the field of medicine and surgery.

"I returned recently from a trip around the world. I visited 17 countries, among them China, Japan, and India. I was amazed when I had the opportunity to observe the great popularity of our motion pictures and, in spite of the fact they had high public acceptance, that there were more pictures made in Japan and India than there are in America. To find this, as I did, is to reveal the imprint the picture has made on the lives and habits of these people.

"I cannot tell you what it means to me to stand here and tell you about this Nickelodeon. But I am reminded of one thing that Philadelphia can take a great deal of pride in and boast about. Philadelphia was the scene of the first motion picture production. There are many firsts that Philadelphians can claim, and for which they have been hon-

ored throughout the world. Here is the city in which liberty was first proclaimed. Here was our first national capital, our first Supreme Court, our first House of Congress. Yes, and here Betsy Ross designed our first national flag. And now we have the Nickelodeon. For the industry I am honored. I feel with pardonable pride, and the industry feels, that it is a great privilege to have The Franklin Institute recognize us and make us a member of its great and illustrious family."

The President of the Institute, in accepting the gift, said that Mr. Goldman had now become a part of the Institute. The Secretary was then recognized and read the citation of the award of life membership to Mr. Charles Laughton, the distinguished actor, in recognition of his contribution to public education through film. Accompanying the award was a memento in the form of a half-dollar piece bearing the head of Franklin (first put into circulation by the government here at the Institute in 1948) mounted on a gold plaque, suitably inscribed.

In acknowledgment, Mr. Laughton addressed the statue of Franklin, saying:

"Mr. Franklin, sir, you dominate us in size and in every way. I think I can say that all the members of the motion picture industry are very grateful that your great and solid Institute has, eventually, made us respectable. I always think, sir, that actors should not give opinions, so I will 'duck' any opinions. I should like to say while I am here that I would like to thank Mr. Goldman on behalf of Mr. Gregory and myself for giving us our freedom to do what we want in the motion picture industry. I hope, Mr. Franklin, that we will never let you down.

"You know, my favorite thing you wrote was in a letter—no, no, not *that* letter, sir—it was a letter you wrote to a lady, a French lady. I think you were making advances to her. You wrote to her that you had dreamt last night you were in the Elysian fields. Your wife had died, the lady's husband had also died. And you dreamt you had met the lady's husband and that he had said, since he arrived in the Elysian fields he had married, and that his wife was out gathering flowers, but she would be back in a few minutes. When the lady came back, she turned out to be Mrs. Franklin, the late Mrs. Franklin. You finished the letter with one glorious line, sir: 'Let us avenge ourselves!'

"Historically, I do not know whether you got her or not, but I hope you did. You deserved her."

Mr. Laughton then moved the audience by reciting Lincoln's Gettysburg Address.

The guests adjourned to the Nickeldeon, where Mr. Goldman and Mr. Laughton made hand and foot prints in cement, and a plaque was uncovered recording Mr. Goldman's generous gift.

On the following day the Nickelodeon was thrown open to Museum visitors and will remain a regular feature of its exhibits. A selection of very early films will be shown daily, and these will be changed at regular intervals.

ONE HUNDRED YEARS AGO IN THE *JOURNAL* *

Thirty-fifth general Report of the Chesapeake and Delaware Canal Company, June 5, 1854

The President and Directors of the Chesapeake and Delaware Canal Company, in making their thirty-fifth annual report to this general meeting of Stock and Loanholders, have pleasure in stating, that the business, during the past year, has been conducted with regularity, and without accident or serious interruption to the constantly increasing trade.

The tow-paths, drains, and bridges have all been extensively repaired and improved, and the works generally are now in better condition for the transit of boats and vessels, than they have been at any time since the opening of the canal.

From the statements of the Treasurer, herewith presented, it will be perceived that the revenue from tolls, for the year ending May 31, 1854, amounts to the sum of	$246,695.02
The contingent and incidental expenses, which include all items for repairs, interest on loans, officers' salaries, wages, and contingent charges, for the same period, amount to	200,131.46
Leaving a surplus of (nett profits arising from tolls,)	$46,563.56

At the last general meeting of the Stock and Loanholders, the necessity for additional and larger locks was communicated to the meeting.

By a resolution then adopted, the Board was authorized to undertake the necessary measures for the accomplishment of that purpose. The Board immediately proceeded to the execution of this important work. Having secured the services of Ashbel Welsh, Esq., an eminent and distinguished Engineer, the surveys and necessary explorations, preliminary to the establishment of the sites for the new locks, were commenced and prosecuted, and after a full examination of the natfire of the ground in which these locks were to be placed, the Engineer recommended to the Board, the location of two locks, one at St. George's and the other at Delaware City, on the south side of the present ones at those points, and the location of a single lock at Chesapeake City, on the north side of the two now at that place, having the same lift as both of them.

After a full consideration of all the circumstances connected with this important work, the Board came to the conclusion, that locks of 220 feet long by 24 feet wide were most advisable, and it was therefore decided to have them constructed of these dimensions. The plans and specifications having been prepared by our Engineer, proposals for the construction of the locks were advertised for, and the contract for them, under the direction and supervision of the Engineer, was awarded to Messrs. Candee, Dodge & Co.

The completion of these locks, though delayed beyond the time expected and contracted for, is now near at hand; the work has been a difficult one, requiring energy, skill, and constant attention to avoid serious accidents and interruptions to the trade of the Canal, but the Board have great satisfaction in being able to state, that they consider the works now so far progressed as to be free from all danger of casualty, and that pending their construction, up to this time, there has been no accident, and that the trade upon the Canal has not been interrupted for an hour.

The results of the year's business have been highly satisfactory—the receipts of the year ending May 31st, 1854, (the largest of any preceding year,) were unnaturally swelled by having not only the usual timber trade of that year, but a great portion of that of the prior year thrown into it, in consequence of the low stage of water in the spring of 1851 in the Susquehanna, which prevented the floating of timber.

Notwithstanding this, and the fact that owing to the very severe winter and late spring of the present year, which has deprived us of the ordinary trade on the Canal during these seasons, the receipts of this year slightly exceed those of the last year.

The Board, therefore, feel fully warranted in saying, that the business of the year just

* Reprinted from pages 163–165 of this JOURNAL, Vol. 59, March 1855.

closed, exhibits the same general improvement, which has marked that of each preceding year; and, in assuring the Proprietors, that, after the completion of the new locks, which will give such important increased facilities, the business of the Company must and will continue to increase from year to year, for many years to come.*

To meet the expenditures for the construction of the new locks, and other expenses necessarily incidental thereto, it was deemed proper by the Board to increase the capital of the Company, by an additional Loan. At separate meetings of the Stock and Loanholders, held at the office of the Company, on the 18th day of July, 1853, after due notice had been given of said meetings and of the purpose for which they were called, the propriety of obtaining the Loan was sanctioned and approved by a majority in interest of the whole number of Stock and Loanholders. A call for proposals for $250,000 of this Loan has been made, and proposals have thus far been received to the extent of $180,000, with every prospect that the balance will be taken by the first of July, before which time the money will not be needed.

ANDREW C. GRAY
President Ches. and Del. Canal Co.

Philadelphia, June 3d, 1854

* Between the opening of the Tide-water Havre-de-Grace trade and the first of June, instant, there have passed between Havre-de-Grace and Philadelphia, through the Chesapeake and Delaware Canal, 50,282 canal boats, without loss, excepting that which occurred to some two or three boats in a towage on the western end of the line, and in one or two other single instances of boats to a small extent.

TO MEMBERS AND THOSE WHO KNOW THE FRANKLIN INSTITUTE

All of us associated with The Franklin Institute are gratified by the support given by its benefactors in the past. Without their generosity our privately endowed Institute would have been unable to serve industry, the professions, and the public as it has and should.

There is still a continuing and pressing need of contributions and bequests —both for general and specific endowment, and of income to be judiciously used. Your response will be most earnestly appreciated.

Checks should be made payable to The Franklin Institute, and sent to The Franklin Institute, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pennsylvania.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

We shall be glad to supply any additional information regarding gifts and memorials. Please write to the above address, attention of the Endowment Department.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT
NICOL H. SMITH, DIRECTOR

AN ISOTHERMAL ANNEAL STUDY OF CARBONYL IRON *

BY
L. MULDAWER AND F. E. JAUMOT, JR.

The staff of the Solid State Physics Division of the Laboratories is continuing the study of the nature and properties of carbonyl iron powders, using General Aniline and Film SF powder.[1] The effects of isothermal anneals on mean crystallite size, strain, coercive force, and the amount and size of Fe_4N precipitate were investigated.

Strain and crystallite size were determined by X-ray diffraction techniques due to Warren and Averbach. It was found that the strain is substantially reduced in these powders before crystal growth occurs. The activation energy observed for recovery was of the order of 20 Kcal/mol. Crystal growth occurs by a second process with an activation energy of approximately 46 Kcal/mol.

Iron powders prepared by the carbonyl process contain considerable impurities (0.5% N, 0.7% C, and 0.5 to 0.8% O), which play an important role in the properties of the bulk powder. Diffraction lines due to iron oxides are observed in the X-ray patterns of the untreated powder, but the carbon present must be widely dispersed as evidenced by the fact that no carbide lines were observed in the X-ray patterns until the powders had been annealed for forty hours at 350° C. On the other hand, Fe_4N precipitates were found early in the annealing process and could be followed in a semi-quantitative fashion. The precipitate size increases with time of anneal and the maximum relative amount of Fe_4N precipitates of measurable size is about the same for all temperatures below 450° C. This maximum amount is reached sooner at higher temperatures and tends to decrease with long annealing times due to the mobility of the nitrogen.

The magnetic coercive force was measured and found to be unexplainably low in unannealed powders. For temperatures below 450° C., coercive force increased with annealing to an equilibrium value; this value was decidedly lower for high temperature anneals or hydrogen anneals, which tend to reduce the amount of Fe_4N present. Finally, a strong correlation between crystallite size and coercive force was found for crystallite sizes smaller than 300 to 500 Å. This correlation held

* This work was supported by the U. S. Atomic Energy Commission.

[1] F. E. Jaumot Jr. and L. Muldawer, Jour. Franklin Inst., Vol. 256, p. 377 (1953).

independently of the anneal employed to obtain the desired crystallite size.

In summary, it appears that the large strains and small crystallite sizes in the untreated powder are due both to the geometry and the nature of the formation process and to the impurities which are present. As the annealing progresses, the impurities agglomerate and this, along with the movement of loosely bound dislocations, tends to reduce the strain. This process has a relatively low activation energy (approximately half that of crystal growth). After the strain has been reduced to relatively low values, the subcrystal walls move to increase the crystallite sizes. This process has a relatively high activation energy which is of the order of the activation energy for self-diffusion. The magnetic coercive force is apparently controlled to a large extent by the crystallite sizes when these are small, but once the crystallite size has increased to 500 Å or so, the controlling factor seems to be the presence of Fe_4N precipitates.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, FOUNDER

Addendum to **Synthesis of Some 2-Oxo-4-Aryl-5-Carbethoxy-6-Trifluoromethyl-1,2,3,4-Tetrahydropyrimidines,** by HENRY A. RUTTER, JR. AND LILLIE O. GUSTAFSON, (JOUR. FRANKLIN INST., Vol. 258, No. 5, November, 1954).

Further proof of structure of 2-oxo-4-(3,4-diethoxyphenyl)-5-carbethoxy-6-trifluoromethyl-1,2,3,4-tetrahydropyrimidine was obtained by comparison of the infrared spectrum* of the compound with that of the 6-methyl analog. The spectra are identical except that a strong band for the C-F bond was found at 8.23 and 8.41 microns in the trifluoromethyl derivative.

More extensive investigation of the ultraviolet absorption spectra of the two compounds indicated the presence of a second absorption peak at 233 millimicrons (density 0.823) for the trifluoromethyl derivative. This may indicate that the compound exists in the enol form in absolute ethanol.

These comparisons of the infrared spectra and of the ultraviolet spectra furnish substantiation of the structure of this trifluoromethyl derivative.

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* Grateful acknowledgment is expressed to Dr. Harold C. Beachell, Department of Chemistry, University of Delaware for arranging infrared examinations of the compounds.

NOTES FROM THE BARTOL RESEARCH FOUNDATION

W. F. G. SWANN, DIRECTOR

RESULTS OF LIMITATIONS ON SPACE CURRENT DENSITIES IN GALACTIC AND INTERGALACTIC SPACE *

BY

W. F. G. SWANN

The high electrical conductivity of space has raised difficulties in mechanisms concerning the production of cosmic-ray energies. High conductivity does not avoid limitation of the current density, i, to the value nec where n is the sum of the numbers of positive and negative ions per cc. Thus for $n = 1$, the saturation current density is of the order 5×10^{-9} ampere/cm^2. The effect of saturation is illustrated by the following idealized example. Consider two coplaner rings of gas A and B, A being the smaller. Under high conductivity, it would be impossible for the magnetic flux through either circuit to change and the electric field would be zero. If B expands, the flux through A due to B changes, but A acquires induced current to compensate the change. As time proceeds, and if the ionic density in A is small, a stage will be reached at which A can no longer develop the current necessary for magnetic flux compensation. An electric field will now arise in A and the ions will become accelerated to cosmic-ray energies, not by appreciable alteration of velocity which has already attained its maximum c, but by alteration of relativistic mass.

* Assisted in part by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

ANGULAR DISTRIBUTION OF THE RESONANCE FLUORESCENCE RADIATION FROM THE 411 KEV LEVEL IN Hg^{198} *

BY

FRANZ R. METZGER

Using the resonance fluorescence effect obtained by heating a source of Au^{198} to 1125° C., the angular distribution of the resonance radiation from the 411 kev level in Hg^{198} has been measured. The experimental distribution has the form $1 + (0.37 \pm 0.06)P_2(\cos\theta) + (1.07 + 0.08)P_4(\cos\theta)$. It agrees very well with the distribution expected for an excited state of spin 2 and a ground state of spin 0.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

GAMMA RAYS FROM PROTON CAPTURE IN Li^7 *

BY

C. P. SWANN, M. A. ROTHMAN, W. C. PORTER AND C. E. MANDEVILLE

Thin targets of metallic lithium have been irradiated by protons supplied by the ONR-Bartol Van de Graaff generator. The excitation function for production of gamma rays of energies greater than ~ 10 Mev has been determined for 1.5 Mev $\leq E_p \leq$ 5 Mev. At 2.1 Mev, a broad resonance of half-width 0.4 Mev is superposed upon a slowly rising continuum. At an angle of observation of 90° with the incident beam, the resonance is pronounced, corresponding to a level in Be^8 at 19.1 Mev. Only a portion of the increased yield at 5 Mev over that at 2.5 Mev can be attributed to the use of a fixed discriminator setting in observing the gamma rays.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

COINCIDENCE STUDIES OF THE DISINTEGRATION OF Pm^{151} AND Nd^{147} *

BY

H. S. HANS,[1] BABULAL SARAF[2] AND C. E. MANDEVILLE

Chemically pure Nd_2O_3 was irradiated by slow neutrons on five successive occasions in the Brookhaven pile. The 27.5-hr. Pm^{151} was found to emit gamma rays of 64, 100, 163, 177, 240, 275, 340, 440, 650, and 700 kev. The 11-day Nd^{147} was shown to emit quanta of energies 92, 165, 280, 320, 410, 440, 530, and 690 kev. The relative intensities of the various quantum radiations have been determined, and coincidence measurements have been performed to ascertain the various sequential relationships between pairs of gamma rays. Partial decay schemes of both radionuclides will be discussed.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] Fulbright Fellow from The Muslim University of Aligarh, India.

[2] On leave of absence from Agra College, Agra, India; at present at the Nobel Institute for Physics, Stockholm, Sweden.

IRIDIUM (194) *

BY

C. E. MANDEVILLE, JAGDISH VARMA[1] AND BABULAL SARAF[2]

Electromagnetically enriched quantities of Ir^{193} were exposed for time intervals ranging from 0.5 to 2.0 hr. on five successive occasions in the Brookhaven pile. The 19-hr. Ir^{194} was found to emit gamma rays of energies 0.295, 0.325, 0.635, 0.640, 0.93, 1.14, 1.28, 1.45, 1.58, 1.77, and ~ 2.00 Mev. By coincidence studies, sequential relationships were established between ten pairs of gamma rays. The first and second excited states of the residual nucleus were located at 325 and 620 kev. The angular correlation function of the 0.295 Mev-0.325 Mev cascade

was measured and found to correspond to a $2 \rightarrow 2 \rightarrow 0$ distribution, thus giving the spins of the first two excited states of Pt^{194}.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] Research Fellow, Bartol; at present at the University of Glasgow, Scotland.

[2] Research Fellow, Bartol; at present at the Nobel Institute of Physics, Stockholm, Sweden.

SHORT-TIME POLARIZATION IN THORIUM OXIDE *

BY

W. E. DANFORTH

When a continuous current of several microamperes per square centimeter is passed through a single crystal of oxygen-poor thorium oxide in vacuum, the potential difference across the crystal rises abruptly at first and then continues to increase in time. Observations between 900° C. and 1300° C. show that, immediately after the initial rise, the rate of rise of polarization is so rapid as to require oscillographic examination. The time required for final equilibrium to be established is, however, several hours. Measurements of the initial rise have been made as a function of temperature. Values of resistivity vary from 5700 ohm-cm at 900° C. to 400 ohm-cm at 1280° C. The plot of $\log \sigma$ *vs.* $1/T$ is linear and yields an activation energy (u) of 2.2 ev assuming the form $\log \sigma = u/2kT$. Ultraviolet absorption spectra of low-oxygen thoria show a maximum at 3.0 ev. The observed conductivities result from the motion of ions as well as electrons. If, as appears probable, the electronic conductivity predominates, the activation energy and the absorption peak may be related to the same band of donor centers.

* Assisted by the U. S. Navy, Bureau of Ships, and by the Office of Ordnance Research.

NEUTRON INELASTIC SCATTERING MEASUREMENTS *

BY

G. W. McCLURE AND D. W. KENT

A low-voltage accelerator of the Cockcroft-Walton type is now providing a prolific source of high energy (14 Mev) monoenergetic neutrons. These are produced by bombarding tritium absorbed in zirconium with a 100 μa beam of deuterons accelerated through a potential of 95 Kev. Owing to its relatively low accelerating voltage and isolated location, the generator is particularly suited to use in investigations requiring a low background of secondary radiation. Measurements are being made of the gamma-rays emitted as a result of inelastic collisions of the 14 Mev neutrons in an iron scatterer. Several discrete gamma-rays evident in the pulse-height distribution obtained with a NaI scintillation counter correspond to transitions between known low-lying energy levels in the Fe^{56} nucleus. There is also evidence of a substantial number of high energy gamma-rays. By means of coincidence techniques, attempts are being made to determine the cross section for

the direct excitation of the low levels as distinguished from their indirect excitation by cascade gamma-ray emission from higher states.

* Assisted by the U. S. Atomic Energy Commission.

COULOMB EXCITATION OF HEAVY NUCLEI *

BY

D. M. VAN PATTER, M. A. ROTHMAN, C. E. MANDEVILLE AND C. P. SWANN

Preliminary investigations in the field of Coulomb excitation of heavy nuclei have been in progress for two months, in conjunction with the large electrostatic generator. The first decisive experiments in this field were made two years ago, and at the present time experimental investigations are being actively pursued at several laboratories. It is possible to excite low-lying states in almost any nucleus by the perturbation caused by the electric field of a bombarding particle, such as a proton, or alpha-particle. The most common method of detecting this process of electric excitation is to observe the gamma-rays emitted when the excited states de-excite, using a NaI scintillation crystal. At our laboratory, the spectra of pulses from the detector are registered on a 20-channel pulse analyzer.

One of the first problems to be faced, if one wishes to use thin targets placed on a backing material, is to choose a backing material which, on bombardment, does not give gamma-radiation which will interfere with the gamma-rays being investigated. With this purpose in mind, targets of nickel and niobium were bombarded with 3.5 Mev protons. Nickel was found to be unsuitable as a target backing for, in addition to several gamma-rays observed during bombardment, at least two activities were found to be induced, due to (p,γ) reactions involving different nickel isotopes. In the case of niobium, two gamma-rays of energies less than 1 Mev were observed, with energies of 0.71 Mev and 0.87 Mev. Thick target excitation curves of both gamma-rays were measured for proton bombarding energies of 2.75 to 4.0 Mev. The excitation curve of the 0.71 Mev gamma-ray agreed with the theoretical curve for electric excitation of a 0.71 Mev level in Nb^{93}, previously unreported. The excitation curve of the 0.87 Mev gamma-ray did not rise rapidly enough to agree with electric excitation. The presence of this gamma-ray is ascribed to the $Nb^{93}(p,\gamma)Mo^{94}$ reaction, since its energy agrees with the first known level of Mo^{94} at 0.874 Mev.

Investigations of thick targets of other elements are in progress, with the objective of looking for excitation of gamma-rays of over 300 kev, which, because of their relatively weak intensity, may not have been observed by previous investigators. At the present time, no gamma-rays in addition to those already reported have been found for targets of tantalum, platinum and osmium. It is also planned, if any new gamma-rays are observed, to determine absolute cross-sections and angular distributions.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

BOOK REVIEWS

ROBERT ESTIENNE, ROYAL PRINTER, by Elizabeth Armstrong. 310 pages, illustrations, 19 × 27 cm. New York, Cambridge University Press, 1954. Price, $10.00.

The admirers of Benjamin Franklin as a scholar and printer should have a special interest in this book. For, if we avoid pushing the resemblance any further, Robert Estienne shared with Franklin the combination of abilities to produce learned works with the energy to run a successful business.

The subject of this excellent book was a member of a family distinguished for its contributions to French printing. Robert Estienne was a noted biblical and classical scholar who became an outstanding figure in the book trade of Paris in the reign of François I. He held the office of King's Printer, the highest honor the Crown could bestow upon one of his craft. This office gave him opportunities to edit hitherto unpublished Greek manuscripts that the king's librarians had collected, and he achieved some spectacular publications. Meanwhile, as king's printer in Latin and Hebrew, he boldly continued to issue editions of the Bible, or portions of it, at a time when the faculty of the University of Paris was ruthlessly persecuting anyone suspected of upholding the principles of Luther and Calvin. Because he became one of the theologians' victims and had to escape to Geneva, Estienne's work as a printer has been overshadowed by the mischief caused by the fruits of his scholarship.

Mrs. Armstrong has performed in admirable fashion the task of producing a balanced biography. She has presented us with a brightly written scholarly work reflecting prodigious research. The figure of the master craftsman is drawn against the background of the golden age of the press, that is, at the time of the Renaissance. During the twenty-five years of his activity in Paris, Estienne published a total of 460 to 470 editions. This probably represented the production of four presses at the time when most printing establishments had but one. But he was not satisfied to have one of the largest printing houses. His object was to have the best. He not only used Roman, Greek, and Hebrew type faces of good design, but he used them with intelligence and good taste. The excellence of his press work, the fine quality of the paper, and the classic proportions of the margins stamp him as a craftsman of unusual qualities. The typography of his edition of Cicero alone would establish him as a printer of distinction.

It is natural that the major portion of the book should be devoted to Estienne's scholarship for, in this respect, he was far in advance of his printing rivals, and to his troubles with the censors. The author has handled her materials with such skill that the reader retains a clear impression of the hazards encountered by scholars, printers, and publishers during those anxious years when Europe was ravaged by religious disputes.

It would be unjust to omit mention of the excellence of the book's typography. The type is the famous Garamond face, the illustrations are numerous, well-chosen, and finely reproduced. From all angles this is a book worthy of the highest recommendation.

T. COULSON

MATHEMATICS AND PLAUSIBLE REASONING: VOLUME I, INDUCTION AND ANALOGY IN MATHEMATICS; VOLUME II, PATTERNS OF PLAUSIBLE INFERENCE, by G. Polya. 280 pages (Vol. I) and 190 pages (Vol. II), 16 × 24 cm. Princeton, Princeton University Press, 1954. Price: $5.50 (Vol. I); $4.50 (Vol. II); $9.00 (the set).

An aura of mysticism has always surrounded the mathematician and his handiwork with the result that the contribution of the individual does not spell out the process by which creative genius operates. This puzzled Professor Polya, who is interested in problem solving and the psychological aspects of mathematical discovery. Having already looked into the former, he now gets around to shed some light on demonstrative

and plausible reasoning which are the prevalent forces in mathematical work.

The first volume describes some aspects of plausible reasoning. The concept of induction is introduced in the first chapter; it is carried over to various fields in later parts of the book: Chapter III, Induction in Solid Geometry; Chapter IV, Induction in the Theory of Numbers; Chapter V, Miscellaneous Examples of Induction; Chapter VII, Mathematical Induction. Three tools are essential in the inductive process; they are discussed in the second chapter, Generalization, Specialization, Analogy. They are further exposed in Chapter VI, where we find a modernized translation of Euler's celebrated memoir on the sum of divisors of natural numbers. The great Swiss mathematician's thought processes are analyzed and some profound and illuminating reflections are made in this chapter on "A More General Statement." Other aspects of plausible reasoning arise in the calculus (Chapter VIII, Maxima and Minima) and in physics (Chapter IX, Physical Mathematics). Finally, the author returns to geometry and develops the various forms of the isoperimetric theorem in the tenth chapter, before adding "Further Kinds of Plausible Reasons" in the last chapter.

This first volume exposes the reader to various aspects of plausible reasoning. He delves into commonly used reasoning processes and finds increasingly numerous grounds to increase his confidence in some particular conjecture. Reasons from analogy, induction, related cases, experience, from the inherent simplicity of the conjecture all combine to make the conjecture very plausible. "Perhaps our confidence in a conjecture is never based on clarified grounds alone; such confidence may need somehow our whole background as a basis."

In the second volume, Professor Polya proceeds to formulate the observed patterns of plausible reasoning explicitly. The first two chapters (numbered XII and XIII for the sake of continuity with the first volume) develop "Some Conspicuous Patterns and "Further Patterns and First Links." The main effort is directed onto establishing logical connections between patterns of plausible inference. To do so, we must extend the domain of pure formal logic to include credibility statements, such as "A conjecture becomes more (or less) credible because of certain observed patterns." This necessary excursion into the foundations of theoretical logic begins with the fourteenth chapter on "Chance, the Ever-Present Rival Conjecture." Statistical concepts are brought to fruition in the climactic fifteenth chapter on "The Calculus of Probability and the Logic of Plausible Reasoning" which provides the final link between heuristic reasoning and probability theory. Well known probability theorems are employed without attaching definite values to the probabilities; the important point is the examination of changes in the probability of a conjecture under the influence of accumulated evidence. A final chapter on "Plausible Reasoning in Invention and Instruction" illustrates the fact that the mathematician does not only guess—he has to find proofs after all.

Two outstanding features in this "Guide to the Art of Plausible Reasoning" should prove attractive to the reader. First, both volumes are written in a simple yet beguiling style, approaching the effectiveness of the spoken word. Second, an abundance of clever problems from a variety of fields stimulates the reader to keep up with the lively pace set by the author. Although these books are intended for use in the class room, they should provide many entertaining hours for anyone who cares to pick up the challenge.

CARL HAMMER

OUR AMERICAN WEATHER, by George H. T. Kimble. 322 pages, 15 × 21 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $4.75.

"In case anybody is by now assuming that the climatologist's world is full of neat little rules of thumb linking behaviour of one atmospheric element here with the behaviour of another there, and a third elsewhere, let me hasten to say that it is. The only trouble about nearly all of them is that they don't tell you very much: and there's little or nothing you can do with them when you've got them." So writes Dr. George H. T. Kimble, an outstanding geographer and meteorologist in *Our American Weather*.

Actually we have not progressed too far from the days when the farmer looked into the sky and from the appearance of the

cloud formations and from the variation in the thermometer and barometer on the back porch decided what the immediate weather was going to be. People in the yesteryear by using rules of thumb and proverbs and other products of our folklore became fairly proficient in the art of forecasting the immediate weather.

There was born a familiarity with the symptoms of change which often reflected the true state of the weather and there were also born weather sayings which while taken for granted simply are completely erroneous.

In a completely fascinating book Dr. Kimble tackles most of the prominent weather sayings and analyzes them for content and truth. Some are completely proven absurd while others are substantiated. As an example of his analysis he takes the saying: "too cold to snow." He shows that the moisture content of the atmosphere is a function of the temperature for this governs the amount of water contained in it. When it gets very cold it is true that the moisture content decreases and so the chance of snowing decreases but does not vanish—it simply cannot get too cold to snow. But the other significant point brought out is that the chance of a very heavy snowfall in extremely cold weather is unlikely if not impossible. The air simply does not contain enough moisture. From this reasoning when snow begins to fall a glance at the thermometer may indicate the depth to which the snow will fall. Dr. Kimble points out that in this country the heaviest snowfall will come when the temperature is between 24° and 30° F. The next time it begins to snow prove this to your satisfaction by watching the variation in temperature.

Little bits of information which are of extreme interest to many peoples are contained therein. For instance, the author speaks of critical temperatures. At 32° snow will melt through the 24 hours of the day. This is common knowledge. But at 43° the plants begin to bud. Gardeners will appreciate this knowledge.

The book is honeycombed with maps of the United States showing when Spring arrives at the various parts of the country. Average dates of the last killing frost in the spring, tornado density, annual precipitation, beginning of summer, average number of days during which thunderstorms occur, first killing frost in the fall, number of foggy days and many other encapsulated bits of information.

The author discusses the appearance of flying saucers as a result of temperature inversions and also deals—albeit not too kindly—with cloud seeding and weather making. The mechanics of smog formation is particularly lucid; the probability of a white Christmas is given (it is not too good—about 1 chance in 4).

One correction to be made in future editions is the differentiation of electrons and radiant energy. While it is true protons are received from the sun we have no definite information as to electrons. However, the energy which the earth uses as the heating medium is radiant energy—photons—but not electrons. Similarly we get radiant energy from the stars in interstellar space but not electrons. This entire section should be looked at by an astronomer or physicist.

Our American Weather is one of the most fascinating weather books available today. Its future as a conversation piece at the dinner table is assured. I. M. LEVITT

AIRPLANE STRUCTURES, by Alfred S. Niles and Joseph S. Newell. Volume I, fourth edition, 607 pages, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1954. Price, $7.75.

Like its predecessors, this reviewer considers this latest edition to be the "Bible" in its field, both for the aeronautical student and the practicing junior engineer alike. Since the basic methods of stress analysis are essentially the same, but the most efficient methods of applying them and the types of structures to which they are applied do change, every chapter has undergone extensive revision and rewriting.

Included in the significant major changes are a new chapter on the physical properties of structural materials, a new method for the analysis of lap joints and increased emphasis and coverage of thin metal construction. Noteworthy throughout are the many illustrative examples and excellent illustrations. Fortunately, no sketchy attempt has been made to cover the whole range of methods which might be used;

rather, only those methods are presented, in a concise and thorough manner, which are fundamental and more widely used.

This edition contains fifteen chapters: (1) General design requirements, (2) reactions, shears and bending moments, (3) beam deflections, (4) continuous and restrained beams, (5) properties of materials, (6) simple bending, (7) torsion, (8) truss analysis, (9) graphical methods, (10) ties, columns and compression panels, (11) buckling effects in beams, (12) combined stresses, (13) connections, (14) deflections, dummy-unit-load method, and (15) introduction to statically indeterminate structures. Three appendices contain useful curves and tables, additional references and answers to problems. SAMUEL M. BERKOWITZ

OPTICS. LECTURES ON THEORETICAL PHYSICS, Vol. IV, by Arnold Sommerfeld, translated by Otto Laporte and Peter A. Moldauer. 383 pages, diagrams, 16 × 24 cm. New York, Academic Press, Inc., 1954. Price, $6.80.

This book, like the other books of the late Professor Sommerfeld, bears the mark of the "master theorist"—Sommerfeld handles theoretical physics like a virtuoso handles the violin, and the reader or listener reaps the benefits. It is a genuine pleasure to read such a book, and although it is written from an advanced point of view, it seems to this reviewer that beginning students (junior and senior physics majors) would acquire a better understanding and certainly a more humble appreciation for physics from such a book than they would from the more elementary and quite often dull textbooks found in colleges.

The first chapter of the book is on Reflection and Refraction of Light; these effects are treated as boundary value problems. This includes a brief discussion of metallic reflection, the classical work of Hagen and Rubens, and remarks on the color of metals, glasses and pigments.

Chapter two is on the Optics of Moving Media and Light Sources. In this chapter are discussed the measurement of the velocity of light, aberration and parallax, Doppler effect and certain other astronomical effects of fundamental importance to the theory of light. A short comment on the quantum theory of light is included.

Chapter three is on the Theory of Dispersion. Included is a detailed treatment of the serious problem posed in the early days of relativity theory; namely, the concepts of phase velocities and precursors having velocities greater than c. This problem was thoroughly treated in 1914 by Sommerfeld and Brillouin.

Chapter four treats Crystal Optics and Chapter five is an advanced treatment of the Theory of Diffraction, including a rigorous solution of the problem of diffraction at a straight edge. Chapter six contains addenda to the theory of diffraction, and includes a discussion of Cerenkov radiation. The book contains a number of problems as well as their solutions. C. A. DOMENICALI

STUDIES IN MATHEMATICS AND MECHANICS. Presented to Richard von Mises by Friends, Colleagues and Pupils. 353 pages, plate, 16 × 24 cm. New York, Academic Press, Inc., 1954. Price, $9.00.

These studies were presented to Richard von Mises as an anniversary volume to commemorate his seventieth birthday on April 19, 1953.

von Mises' fields of interest ranged from literature and philosophy to statistics and mechanics. He made important contributions in many fields. For example, he is considered an authority on the poet Rainer Maria Rilke. However, his official field is considered to be applied mathematics and mechanics. Hence this volume is devoted principally to this field.

The volume contains 42 studies in mathematics and mechanics, divided into the following five subdivisions: (1) Algebra and number theory; geometry. (2) Analysis. (3) Theoretical mechanics. (4) Applied mechanics. (5) Probability and statistics.

The languages used in these papers are English, French, Italian and German. There are occasional misprints. However, since the volume is addressed to specialists, these should cause little difficulty.

H. L. PLATZER

CONTROL-SYSTEM DYNAMICS, by Walter R. Evans. 282 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $7.00.

Determining the response of a linear control system is a fundamental step in the design of a servomechanism. There are several established methods for such an analysis, each having advantages and disadvantages. In the control-system problem considered in this volume of the McGraw-Hill Electrical and Electronic Engineering Series, the designer is asked to select parts of a dynamic system, which, when used in conjunction with preassigned components of the system, will yield an over-all system's response of the desired kind. The forte of the author is the so-called root locus method for factoring the algebraic polynomials which arise in analyzing the pertinent differential equations which describe the performance of feedback control systems.

The root locus method was developed by the author and his colleagues at North American Aviation, where it has been applied to many control problems. The book itself emphasizes a physical understanding of basic problems as contrasted with memorizing a routine for solving particular problems.

Throughout its reading, this reviewer was mildly disturbed by the fact that the text was indefinably unpolished—there seemed to be no question but that the author was a "practicing practical" engineer, unfettered by precision of definition and such other niceties one often finds in books written by practicing teachers of engineering subjects. In the hands of a teacher of control-systems engineering, who would selectively filter, change and add text material, the book should be a most useful addition to others in this expanding field. It can stand on its own merits as a reference book.

Most of the subject matter of the twelve chapters and seven appendices is well selected. The first seven chapters cover basic material such as the frequency-response method of analysis, the root locus method, simple servomechanism components, the fundamentals of transient response and transfer functions. The remaining chapters are devoted to multi-loop systems, circuit analysis, the determination of the transfer function and transient response, and graphical analysis of quadratic systems made nonlinear by such effects as Coulomb friction, pick-off dead zone and backlash of gears.

The appendices, a short bibliography and sixty problems, half of which require an extension of the theory presented in the text for their solution, close the book. In spite of its mild shortcomings, attributed primarily to the inclusion of unnecessary material, this reviewer is of the opinion that this book will be a definite contribution to the field of servomechanism design. It is recommended to practicing electrical and mechanical engineers who encounter control-system problems.

S. CHARP

LAMPS FOR A BRIGHTER AMERICA, by Paul W. Keating. 246 pages, diagrams, plates, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $4.00.

The title of this book does not indicate its full purpose. The subtitle says it is a study of the "history of the General Electric lamp business," and that is what it amounts to. The work of the many pioneers in the field is sketchily dismissed in order to arrive at the work of Thomas Alva Edison, after which few details are lost from sight. The author has delved deeply into the company's archives. In fact the general reader may complain that he has delved to excess. Mr. Keating has organized such an imposing array of facts concerning the management, administration, research, and manufacturing activities that have contributed to raise the electric light to its present stage of efficiency, that we often cannot see the forest for the trees.

For centuries man had to be content with the feeble glimmer of his oil and grease lamps to dispel the deepest shadows of night, but today our indoor lighting is the competitor of sunlight. Not only does the electric lamp furnish us with light, but it provides the radiant energy which makes for healthier living. The General Electric Company has been foremost in contributing to the progress from Edison's carbon filament lamp to the variety of lamps now available on the market. The company deserves to have a record of its achievements.

The author has performed a truly remark-

able piece of work in drawing together the many loose ends of the story. He has overlooked nothing in his search for detail. The scientist, the engineer, the student of business management, and the salesman will be able to turn to this work for information, but each will find the interpretative material missing. The reader will feel he has been admitted behind the scenes of a great company's activities, for hardly a change in the senior personnel can have been overlooked. A great many worthy men have been drawn from the anomymity that shrouds most workers in a firm of this size.

The development of electric lamps for special purposes in recent years has been so prolific in results that the treatment it could receive in a work of this size was necessarily sketchy but, again, the author does not appear to have overlooked any significant feature.

The book will serve as a handy guide to a complicated business structure and to its activities over a period of seventy-five fruitful years. T. COULSON

DETERIORATION OF MATERIALS, edited by G. A. Greathouse and C. J. Wessel. 835 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1954. Price, $12.00.

The authors of this large book are Director and Assistant Director, respectively, of the Prevention of Deterioration Center, Division of Chemistry and Chemical Technology, National Academy of Sciences, National Research Council. A Foreword has been written by Dr. Vannevar Bush.

The first three chapters discuss the three main factors in deterioration, namely, climate, chemical and physical agents, and biological agents.

The second part of the book, chapters four through ten, discusses the deterioration of all the ordinary kinds of materials, such as metals, wood, paper, textiles, leather, plastics, rubber, paints, varnishes, etc.

Part III covers electronic and electrical equipment, optical instruments and photographic equipment.

Part V treats the problem of preservation, including dehumidification, packaging and toxicological evaluation of preservatives. There is an extensive index, and also an appendix on sources and identifying symbols of government specifications.

The book contains contributions from twenty-four experts. It seems silly to say that the book is valuable, for this is obvious from a quick look at the table of contents; but it is not silly to say that the book is also very interesting and valuable even to the casual reader who wonders why the very old books in the library are "rotting away." C. A. DOMENICALI

DEVELOPMENT OF THE GUIDED MISSILE, by Kenneth W. Gatland—Second edition, 292 pages, illustrations, 15 × 22 cm. New York, Philosophical Library, 1954. Price, $4.75.

This book was first published in 1952 and at the time proved to be a significant step in the unfolding of the guided missile story. It contained the essential elements of the new armament and progressed through rockets, space-satellite vehicles and concluded with a short section on interplanetary flight.

However, gargantuan strides have recently been made in this field. Since the beginning of guided missile development in the United States at least $5 billion has been spent on research and hardware. And the effort will be intensified in the future because of the desperate competition in the building of the "ultimate weapon."

Whereas this country has spent in excess of $5 billion on guided missiles it is difficult to estimate what this has cost the world though it is likely to exceed this government's outlay. This tremendous cost has projected the subject of guided missiles into spectacular prominence.

The author has just about doubled the size of the book and has filled it with pertinent information on guided missiles throughout the world. The contributions of this country are conspicuous by the wealth of detail which the author has combed from unclassified sources. Most of this information this writer has seen in the literature but here, at last, it is found within the covers of a single book.

Three appendices are incorporated in the new book compared to one in the first edition. The first is devoted to telemetering systems. The second is a ten-page picture

spread on guided missiles of all nations and the third is the section on "significant missiles." This last was present in the first edition but it has been enlarged to take into account the latest developments.

Development of the Guided Missile is a must in the libraries of those who would like to keep abreast of the latest developments in this field. I. M. LEVITT

TECHNICAL MATHEMATICS, by Harold S. Rice and Raymond M. Knight. 748 pp., 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $6.50.

Wentworth Institute in Boston is one of the technical high schools whose mission is the training of superior students who possess the talents and interests which lead them toward scientific and engineering careers. This book was prepared by two mathematics professors at Wentworth Institute. It is a comprehensive treatment of mathematical subjects suitable for entrance into the engineering colleges. The most advanced work covered by the book provides the student with working experience in complex notation and vector algebra.

Consisting of a relatively large number of pages for a high school text, the book is designed to cover in a single volume algebra, trigonometry, applications of geometry and all the interrelated theorems. There are about 5000 problems in all.

The authors' desire to provide a fairly complete book of instruction is indicated by the inclusion of supplementary material such as a manual of slide rule operation, a chapter on the treatment of measured data, principles of interpolation, and a number of numerical tables. The slide rule manipulations for solving right triangles are taught at the same time the trigonometric relations are considered. Certain of these drills would not be harmful to electrical engineering students who often appear somewhat uncertain and none too dexterous in the use of their large slide rules beyond the A and B scales.

Perhaps the only novelty ascribable to this volume is the combination of the several high school mathematical subjects under a single cover, and possibly the wisdom exercised in selecting the supplementary material. Judging the text on its merits as a foundation for college work, it appears entirely adequate, provided no prerequisite of elementary calculus is to be met. Also, college students may find this extensive volume a convenient reference to which they may refer for forgotten items during the course of their more advanced curricula.

C. W. HARGENS

CURRENT TOPICS

First Transatlantic Telephone Cable.—The first transatlantic telephone cable—a joint Anglo-American-Canadian venture that will provide a clear, reliable, multiple-channel telephone hook-up between the two continents—is now under construction. It should be completed next year, and is scheduled to be in operation by Christmas, 1956.

The aim is transatlantic telephone calls of clear and constant quality, instead of the present "hit-or-miss" system which is dependent on good radio conditions. Also more than twice as many simultaneous conversations will be possible under the new system. Need for the cable is evidenced by the increasing traffic: in 1927 there were a mere 2000 calls; in 1951 there were nearly 100,000.

The new communications system will be a cooperative project of Britain's General Post Office, American Tel. and Tel., and the Canadian Overseas Telecommunications Corp. It will provide 36 telephone circuits between Oban in Scotland and Clarenville in Newfoundland, and from there to Sydney Mines, on the Canadian mainland.

It will require the laying of two undersea cables, each 1950 (naut.) miles long, and of a 335-mile single cable, part overland and part underwater. One hundred and eighteen "repeaters," or boosters, containing 300 valves and 6000 other electrical components, will amplify the sound at regular stages along the route, so that the voice will be as clear at the outcoming as at the ingoing end.

The story of the making of the cable and the boosters, and of the difficult laying operation, is a saga of engineering determination, skill and imagination.

To effect this direct link, which will give more reliable and wider service than the radio-telephone that has provided communication for the last twenty-eight years, has long been the dream and hope of technicians on both sides of the Atlantic. But a number of scientific and engineering problems had to be surmounted before the scheme could be realized.

The discovery by Imperial Chemical Industries in 1933 of a substance named Polythene, and the application of this as an insulant, helped pave the way. Polythene has certain advantages over gutta percha and Paragutta, formerly used to insulate submarine cables, and is considered to be ideal for this purpose.

The new cable now being made will use 1400 tons of polythene. It will also require, for its 1950-mile length, some 2700 tons of copper, 11,000 tons of steel wire, 1800 tons of jute yarn, and 2.4 millions yards of cotton cloth.

Britain will pay 41 per cent of the cost of the $35 million project, the United States 50 per cent and Canada 9 per cent.

One problem that had to be solved before the telephone hook-up could be planned was the development of suitable repeaters, or boosters, to amplify the sound at regular intervals along the line. Here, the partnership between the U. S. and Britain proved its value.

The British, who had experimented for years with co-axial cables, and had

laid them successfully between the Isle of Man and Anglesey, and across the North Sea to Norway, had concentrated on shallow-water repeaters. These had the advantage of being large enough to carry two-way speech transmitters—but had the disadvantage of being housed in large, rigid steel containers.

The Americans, on the other hand, had turned their energies to the development of deep-water repeater housings, and Bell Telephone Labs had evolved and laid between the U. S. and Havana cables with boosters housed in an ingenious type of flexible container resembling a steel snake, which may be coiled inside storage tank drums. The disadvantage of the flexible type of repeater-housing is that it is not large enough for a two-way cable, but can contain electrical equipment for only one-way conversations. Consequently, wherever this is used—all across the deep-sea stretch of the Atlantic—two cables must be laid.

The whole of the first 1500-mile deep-water section will have to be laid in a single operation without pause—for the cable may buckle if the cable ship has to stop during the laying. For this reason it must be done in summer, when a two-weeks stretch of consecutively good weather is a possibility. The first cable will therefore be laid this coming summer; the second in the summer of 1956.

The advantages of the new cable are the widening and improvement of the service, and the possibilities it opens up. At present, no more than 14 conversations are possible at one time, and interference often makes reception poor or stops it entirely. Not only can 36 clear telephone calls be made simultaneously over the new line, but once it has proved itself and with production at the new cable plant rolling, there is no reason why further cables should not be laid if demand increases. It also appears to presage the laying of a transatlantic television circuit at some future date.

Antibiotics Decrease in Price.—Speaking recently at a meeting of 125 key executives of Parke, Davis & Company from the U. S. and Canada, John L. Bach, director of press relations for the American Medical Association, said there was a need for a long-range educational program by drug manufacturers to acquaint the public with the outstanding results in antibiotic development.

He said: "For the patient, antibiotic drugs are a bargain at any price. Antibiotics have not only cancelled out many of the once-fatal diseases, but they have eliminated entirely many of the bread-and-butter diseases which kept doctors so busy years ago.

"Five dollars worth of penicillin can eliminate the need for $150 mastoid operation and $200 in hospital bills," Mr. Bach said, and then added: "Does the public know this?"

The A.M.A. spokesman contrasted the treatment of pneumonia today with the problem faced by the horse and buggy doctor not so many years ago.

"When the old family doctor was called to see patients with pneumonia, he was forced to sign 33 death certificates out of every 100 patients seen. In addition to the number of deaths caused by pneumonia in that day, there was incalculable man-hours lost by prolonged illness and convalescence, which that wonderful old family doctor was powerless to overcome. The cost of medical care for pneumonia patients then was not only high, but the cost in lives and in man-hours lost in prolonged convalescence was tremendous indeed.

"The antibiotics changed this picture completely. Out of every hundred

pneumonia patients seen by a doctor today, he will sign not 33 but only one or, at the most, two death certificates. The man-hours lost in convalescence have, for the most part, been largely eliminated."

Mr. Bach said that "when a life is at stake, the cost of any drug or drugs should be a minor consideration.

"Compared with early prices," Mr. Bach said, "antibiotic prices today are inordinately cheap. In 1944, penicillin cost $20 for an average dose of 100,000 units. Today, it is anywhere from two to eight cents for the same amount. Streptomycin came on the market at $15 a gram. Today, the same amount can be bought for 15 cents. Some of the more recently developed antibiotics in the 'broach spectrum' group have been cut 50 per cent in price.

"Is the public fully aware of this downward trend?" Mr. Bach asked.

He urged all pharmaceutical manufacturers to "tell the public your marvelous story of antibiotic development."

He said that if all drug manufacturers create a better understanding of what they are doing, how they do it, why they do it—and make clear who profits thereby—the cumulative effort will be irresistible. The situation, amid today's cry of high prices, demands the best efforts of the industry. If the industry makes the effort, and attains at least part of the objective, much will be accomplished in erasing the common belief that drugs generally are too costly.

Jet Fighter Launched from Truck.—The U. S. Air Force has announced that in recent experiments piloted jet fighter planes have been sent aloft from launching platforms mounted on trucks.

The technique of launching conventional jet fighters without preliminary takeoff runs was developed in tests at Edwards Air Force Base, California, conducted by the Air Force's Air Research and Development Command and the Glenn L. Martin Company of Baltimore, Maryland.

The launching platform is the same used normally in launching the Martin Matador, an Air Force guided missile. Republic F-84 Thunderjets were modified so that booster bottles such as are used in launching the Matador could be attached beneath the tail.

Highly mobile trucks with "arms" raised the fighter planes to the launching angle, becoming in effect the world's smallest airports. With the planes' turbo-jet engines running at full speed, the thrust of the booster bottles kicked the fighters off so swiftly they were immediately airborne.

Martin test pilot Bob Turner said the shock of the unconventional takeoff was less than that which pilots normally experience during catapult takeoffs. The planes were always under the pilots' control and a peak acceleration of four Gs was reached. One G is the pull of gravity at sea level.

This development is another step in the Air Force's research and development program to explore the possibility of eliminating the requirement for runways under certain combat conditions.

The technique of zero-length launching has been proved also by hundreds of Matador launches by Martin engineers working with the Air Force's Air Research and Development Command and Tactical Air Command.

Structure of Chromosomes.—The building blocks of chromosomes are tiny shreds or fibrils about 25 millimicrons thick, according to the theory of a University of Wisconsin cytologist who reported his research findings to fellow scientists attending the annual meeting of the American Association for the Advancement of Science.

The cytologist is Prof. Hans Ris of the Wisconsin zoology department. The theory may provide the initial step in answering an elementary—but hitherto completely puzzling—question concerning the units in the cell which transmit parental characteristics to offspring.

Until recently no scientist had any but the most general notion of the internal structure of chromosomes.

The findings which led to Prof. Ris' theory were made with the electron microscope at the University of Wisconsin. The microscope showed that chromosomes are made up of many tiny fibrils coiled like a corkscrew and about 25 millimicrons thick or about one 25 millionth of an inch.

During cell division, chromosomes split lengthwise and each half goes to a daughter cell—furnishing the master-pattern which the daughter cells follow in growing into a likeness of the parent. By this means, old cells of tissues and organs are replaced with new young cells identical in structure and function. Division is also the means by which sperm cells and ova are created, each containing the chromosomal master-pattern of a parent, which merge to produce an offspring with characteristics of both parents. During the process of cell division, the tiny fibrils apparently enlarge until they are twice as thick—or 50 millimicrons—and then split lengthwise, thus giving rise to two new fibrils, Ris has found.

Scientists recently have found out a great deal about the chemical composition and properties of chromosomes, but Prof. Ris' effort to describe their internal structure will help enormously to explain how chromosomes are put together and why they behave the way they do. Professor Ris has found that the chromosomes in many different types of plant and animal cells all have fibrils of the same width as basic units.

The work to prepare chromosomes for study under the electron microscope is particularly exacting. They must be removed from living cells, yet not destroyed or distorted. Professor Ris has developed successful methods for chromosome preparation which make it possible to look at them in the electron microscope at magnitudes of many thousand times, and to see details of their inner structure.

New Supporting Blocks for Laboratory Setups.—One answer to the problem of what to use for sturdy, safe laboratory supports and for building up equipment in laboratory setups, has been given by Central Scientific Company with the recent development of a line of handy, various-sized Transite and insulation blocks.

Called Cenco Lab Blocks, they are heat and chemical resistant and are useful for supporting light or heavy pieces of laboratory equipment such as beakers, flasks, hot plates or ring stands. They also provide a safety factor as they give much sturdier support than often used inverted beakers, books, match boxes and other precarious devices which frequently result in costly breakage.

The Lab Blocks are uniformly shaped in 4- and 8-in. squares and come in thicknesses ranging from ¼-in. to 2 in. A set of the blocks enables the laboratory worker to select a wide variety of positions from ¼ to 7¾ in. The blocks come in two different sets. Set Number 1 consists of four blocks, 4 in. square with thicknesses of ¼, ½, 1½ and 2 in. Set Number 2 contains four 8-in. squares measuring ¼, ½, 1½ and 2 in. thick.

"Picture-on-the-Wall" Television?—A long-sought scientific goal—direct amplification of light without use of electronic tubes—was actually demon-

strated recently by scientists from the General Electric Research Laboratory, Schenectady, N. Y.

"Discovery of this light amplifier may be the clue to achieving 'picture-on-the-wall' television screens," according to Dr. C. G. Suits, GE vice president and director of research.

Although the new scientific phenomenon may have great importance in the future of television, X-ray fluoroscopy, photography, "seeing-in-the-dark" devices, and other developments involving reproduction of picture images, Dr. Suits emphasized that no immediate application should be anticipated.

"The importance of today's demonstration lies in the new scientific knowledge involved in achieving light amplification for the first time in a simple phosphor film," he said. "GE's experimental screens, made of a special phosphor to which an electric field is applied, have given off ten times as much light as that projected on them."

In the demonstration, scientists increased the brightness of a projected photograph by passing an electric current through a special phosphor cell used as the viewing screen.

Dr. F. E. Williams, head of light generation studies at the GE Research Laboratory, said the light-amplifying phosphor was created by D. A. Cusano, young GE scientist who conducted the demonstration.

An ordinary lantern-slide projector was employed, and the picture was a regular photo slide. The small screen reflected a yellowish image when an ultraviolet light source was used in the projector.

As the voltage on the screen was increased, the picture became brighter in the manner usually achieved either by increasing the intensity of the projector light or by opening the lens aperture. Neither the light nor the lens was altered, however, and the picture—which at first was barely visible—became many times brighter and clearer.

During the over-all brightening of the projected picture, the contrast remained virtually unchanged, thus demonstrating proportional amplification.

Dr. Suits pointed out that this demonstration was of a new scientific phenomenon, but because of the small size of the screen (about four inches across) and the limitations in the type of light that can now be amplified, no immediate applications for picture projection screens should be anticipated.

"There may eventually be many practical applications of the light amplifier, but it is no more possible to predict these with certainty now than it would have been to foresee the extent and ramifications of modern radio and television when the first electronic amplifying tube was invented fifty years ago," Dr. Suits said.

The demonstration showed that applying an electric voltage does not of itself cause the special phosphor screen to give off light. Ultraviolet energy falling on the screen causes a faint glow, but there is no amplification. Amplification occurs only with the combination of both the voltage and the ultraviolet.

Proof of true amplification is obtained by measuring the number of photons of light striking the screen and comparing this with the number given off. Increases of at least ten times have been measured, and the GE scientists believe much higher ratios are possible. Because the amount of light reflected is proportional to that striking the surface, it is possible to brighten the intermediate shades of the picture being projected, thus making a brighter picture without "washing-out" the contrast.

"The direct light amplifier is an outgrowth of extensive GE research in

electroluminescence, a field devoted to direct conversion of electrical energy into light," Dr. Williams said.

"Modulation of photoluminescence by an electric field has been previously reported by several scientists both in the U. S. and abroad, but true amplification has now been achieved," he added.

Dr. Williams emphasized several years ago that, on the basis of theoretical studies, light-amplifying phosphors are feasible. The success of Cusano's subsequent work provides striking confirmation of the theory.

Method Found to Eliminate Man-Made Fog in Alaska.—Successful Alaskan tests on experimental ice-fog eliminators recently completed by personnel of the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Va., mark a definite advance in the solution of the ice-fog problem at air bases in the interior of Alaska.

The man-made fog results from the discharge of exhaust gases from combustion devices into the atmosphere at temperatures from −24° to −40° F. The low visibility caused by the fog, which at times completely "socks in" the northern bases, seriously hampers aircraft operations.

Field tests were conducted by ERDL on ice-fog eliminators incorporated into two 6-hp. engines. Without the eliminators, fog appeared as a plume 60 to 70 ft. down wind from the exhaust of the two engines. Only slight traces of fog were visible when eliminators were used.

Developed by the Armour Research Foundation under contract with the Laboratories, the eliminators condense the moisture from the engine exhaust gas by cooling, mix it with cold ambient air, and finally reheat the mixture and discharge it to the atmosphere fog free. Equipment based upon this principle can be applied to engine powered ground equipment.

New Tool for Probing Ocean Bottoms.—A new instrument which provides scientists with a method of measuring the compactness of underwater sediments without disturbing them has been developed by scientists at the University of Rhode Island under contract with the Office of Naval Research and the Navy Hydrographic Office. The new instrument, a penetrometer, consists of a steel tube with a probe on the end which is driven through a hollow shaft into the bottom by a motor and a mechanism which measures and makes a permanent record of resistance at depths up to 200 ft. The working mechanism is mounted on a five-foot frame resembling a bell buoy. It weighs 145 lb. without the lead weights which hold it in position under water. The motor which drives the probe is equipped with a watertight cover which can be pressurized. In addition to probing the ocean bottom, the penetrometer may be used on land as a soil-mechanics instrument for highway planning and foundations.

Comfortable Life Preserver That Works.—Since World War II, considerable progress has been made in the development of survival equipment through research and development. The life preserver is one of the most important items of abandon ship equipment aboard naval vessels. The inflatable belt-type life preserver, used during World War II, was comfortable to wear aboard ship, but was too often found useless in the water. It offered no head support to an unconscious man and required positive effort on the part of the wearer to maintain an upright position. A modified British Ad-

miralty type CO_2 inflatable life preserver has been developed and adopted as standard for U. S. Naval shipboard use. Stowed in a pouch and carried on the small of the back, this preserver does not restrict the wearer from carrying out his normal duties. It also has the advantages of being quickly and easily donned, and when inflated, will automatically turn the survivor on his back and keep his head well out of the water.

G-E Scientists Make Diamonds in the Laboratory.—Man-made diamonds, the climax to a 125-year effort to duplicate nature's hardest and most glamorous substance, were displayed last month by scientists of the General Electric Research Laboratory. The tiny diamonds were made from a carbonaceous material subjected to extreme pressures and temperatures. The largest stone measured $\frac{1}{16}$ of an inch in longest dimension.

The hardest substance known to man, diamonds are mined at the rate of about $2\frac{1}{2}$ tons annually. Approximately 90 per cent are imported by the United States. Of the diamonds of industrial grade, some are stockpiled for defense purposes, but the major portion goes into tools that cut, saw, or polish other hard materials. For industrial diamonds purchased in a typical year, the U. S. paid more than $50,000,000.

According to Dr. A. L. Marshall, manager of the Laboratory's Chemistry Department, the G-E diamonds were positively identified by X-ray inspection, chemical examination, and hardness tests. Under hardness tests, the G-E diamonds proved capable of scratching anything, even other diamonds. Thus the G-E diamonds are the first man-made substance to scratch other diamonds.

Making diamonds was the result of more than four years of intensive research by Laboratory scientists under supervision of Anthony J. Nerad, manager of the Mechanical Investigations Section. The scientists responsible for the project were Drs. Francis P. Bundy, H. Tracy Hall, Herbert M. Strong, and Robert Wentorf.

In Dr. Strong's process, a carbonaceous compound was subjected for many hours to a measured pressure some 53,500 times greater than atmospheric (roughly 800,000 psi.). That run, in the Laboratory's new 1000-ton press, yielded a crystal nearly a sixteenth of an inch in longest dimension.

"After opening the pressure chamber," said Dr. Strong, "and while attempting to polish the now solidified matrix, there appeared what we were seeking, a core of superhard matter that refused to wear away under the action of the polishing wheel. With considerable excitement and expectation I removed the crystal from its surroundings, and tried one of its sharp points on sapphire, silicon carbide, and boron carbide. This 'diamond,' I could call it now, easily scratched all three."

Of fundamental importance was the work of Dr. Hall, who extended Bundy's initial high-pressure work and developed the "belt," a chamber enabling G-E scientists to maintain for the first time temperatures above 5000 F. at pressures in excess of 1,500,000 psi.

This pressure is roughly equivalent to the "squeeze" physical scientists have computed for points 240 miles beneath the earth's surface. Using this equipment, late last year Dr. Hall developed a process that produces diamonds in a matter of minutes. Hall's diamonds are smaller, but are often produced in solid clusters. Single runs have produced up to $\frac{1}{10}$ of a carat.

Dr. Wentorf likewise had success

with the same physical apparatus employed by Hall, though different chemical conditions were present. Since these achievements, several new methods have been tried, and these also produce diamonds.

To confirm the results two independent teams of scientists from other departments in the Laboratory have repeated the experiments of Hall and Wentorf, in each case with success. All in all, the various processes for making diamonds have been repeated successfully more than 100 times.

Many of the first diamonds produced were deliberately destroyed during essential testing. For example, many crystals had to be burned to determine their carbon content.

Dr. Marshall, in the course of discussing laboratory tests for diamonds, explained that X-ray examination is perhaps the most conclusive single test, because every crystal from common salt to precious jewel has its own distinctive X-ray "fingerprint," or diffraction pattern. It is easy to recognize the pattern of diamond because no other known material comes anywhere near matching it.

General Electric attaches great importance to the possibility of producing diamonds as a logical extension of its interest in hard cutting materials represented by cemented carbides produced in its Carboloy Department.

The true status of previous attempts to make diamonds is difficult to determine with certainty. Principal claimants of success were a Frenchman, C. Cayniard de la Tour (1828); an English experimenter by the name of Hannay (1880); the French scientist H. Moissan (1894); and a British scientist, Sir Charles Parsons (1907–1920), who subsequently expressed doubt about his own results.

Repeated attempts by many researchers to reproduce the results claimed by these earlier investigators have failed, and careful analyses of existing records by reputable scientists have not confirmed any previous claims that diamonds were actually produced in the laboratory.

Certainly no sure method of repeatedly making diamonds has ever been announced before.

In perfecting the long series of laboratory improvements in their struggles to achieve combined high pressure and high temperature, G-E scientists gained valuable help from nature, particularly from studies of diamonds found in Arizona meteorites.

Knowledge of pressures reached was gained by observing changes in the electrical resistance of bismuth, thallium, cesium, and barium-phenomena discovered by Dr. Percy Bridgman of Harvard University, world-renowned pioneer in high-pressure research.